

Your PRINTED Name is: _____

Please CIRCLE your section:

R01	T10	26-302	Dmitry Vaintrob
R02	T10	26-322	Francesco Lin
R03	T11	26-302	Dmitry Vaintrob
R04	T11	26-322	Francesco Lin
R05	T11	26-328	Laszlo Lovasz
R06	T12	36-144	Michael Andrews
R07	T12	26-302	Netanel Blaier
R08	T12	26-328	Laszlo Lovasz
R09	T1pm	26-302	Sungyoon Kim
R10	T1pm	36-144	Tanya Khovanova
R11	T1pm	26-322	Jay Shah
R12	T2pm	36-144	Tanya Khovanova
R13	T2pm	26-322	Jay Shah
R14	T3pm	26-322	Carlos Sauer
ESG			Gabrielle Stoy

Grading 1:

2:

3:

1. (36 points) Start with the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ 3 & -3 & 7 & 0 \end{bmatrix}$$

- (a) Find a basis for the column space $\mathbf{C}(A)$.
- (b) Find a basis for the null space $\mathbf{N}(A)$.
- (c) Find a basis for the row space $\mathbf{C}(A^T)$.
- (d) Write the complete solution to $Ax = b$.

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ 3 & -3 & 7 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

2. (32 points)

- (a) Suppose the matrices A and B have the same column space. Give an example where A and B have different nullspaces — or say why this is impossible.
- (b) Again A and B have the same column space. Give an example where A and B have different ranks r — or say why this is impossible.
- (c) CIRCLE TRUE or FALSE:
If B is a square matrix then $\mathbf{C}(B) = \mathbf{C}(B^T)$.
- (d) If the columns of a 5 by 3 matrix M are linearly independent and x in \mathbf{R}^3 is not the zero vector, then you know that Mx is _____.
I am looking for an answer that uses independence of columns and $x \neq 0$.

3. (32 points)

- (a) Find a 3 by 3 matrix A whose column space is the plane $x + y + z = 0$ in \mathbf{R}^3 . (This means: $\mathbf{C}(A)$ consists of all column vectors (x, y, z) with $x + y + z = 0$.)
- (b) How do you know that a 3 by 3 matrix A with that column space is not invertible?
- (c) Does there exist a matrix B whose column space is spanned by $(1, 2, 3)$ and $(1, 0, 1)$ and whose nullspace is spanned by $(1, 2, 3, 6)$? If so, construct B . If not, explain why not.
- (d) Is this set of matrices a vector space or not? All 3 by 3 matrices with the column vector $(1, 1, 1)$ in their column space. YES or NO with a reason.

Scrap Paper