18.06 Spring 2013 – Problem Set 2

This problem set is due Thursday, February 28st, 2013 at 4pm (hand in to Room 2-255). The textbook problems are out of the 4th edition. A correct answer will only earn you half of the available points. The other half of the points come from your explanation.

Note: Your recitation instructor is responsible for allowing late homework submissions, as well as the re-grading of your PSet. If there is any problem with your PSet, contact your recitation instructor!

1. (8 pts) Do Problem 6 & Problem 14 from Section 3.2.
2. (8 pts) Do Problem 15 & Problem 18 from Section 3.2.
3. (8 pts) Do Problem 4 from Section 3.3.
4. (8 pts) Do Problem 28 from Section 3.3.
5. (8 pts) Do Problem 3 & Problem 5 from Section 3.4.
6. (8 pts) Do Problem 15 & Problem 17 from Section 3.4.
7. (8 pts) Do Problem 1 & Problem 9 from Section 3.5.
8. (8 pts) Do Problem 16 from Section 3.5.
9. (18 pts) Suppose two subspaces of $\mathbf{R}^n$ have $\dim(V) + \dim(W) > n$. Show that some nonzero vector is in both V and W.
10. (18 pts)
    (a) Find a basis for the space V of 3×3 matrices that have zero row sums:

    $$A_{11} + A_{12} + A_{13} = 0, A_{21} + A_{22} + A_{23} = 0, A_{31} + A_{32} + A_{33} = 0.$$

    Then show that $(1, 1, 1)$ is in $N(A)$ for any matrix A in V. Describe also a basis for the subspace W of 3×3 matrices with zero column sums.

    (b) What is the dimension of the intersection $V \cap W$ (which is all 3×3 matrices A that have zero row sums AND zero column sums) ? HINT: If you know a 2×2 submatrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in the upper left corner of A, what are all the other entries ?

    (c) Hyperchallenge (not graded): Find a 3×3 matrix that cannot be written as $M + N$, where M is a matrix in V, and N is a matrix in W (You should suspect that you can find such a matrix – Problems 3.5.43-44 in the book say that V and W together only span an 8-dimensional subspace and the 3×3 matrices are 9-dimensional).