Pset #2 Solutions

2.6 #13 We can guess the following decomposition:

$$L = \begin{bmatrix} 1&0&0&0 \\ 1&1&0&0 \\ 1&1&1&0 \\ 1&1&1&1 \end{bmatrix} \qquad U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

2.6, #24 Suppose A=LU for an invertible A; then A_k = L_k U_k, where A_k denotes the k x k matrix formed by looking at the entries which lie in both the first k rows and the first k columns (and similarly for L_k and U_k). But clearly L_k and U_k are invertible, since they have non-zero diagonal entries (since L and U are invertible); hence for each 1 <= k <= n, A_k is invertible, i.e. has non-zero determinant.

2.7 #3 Since (AB)^T = B^T A^T:
(Ax)^T y = x^T A^T y = x^T (A^T y)

2.7 #40 (a) Consider the equation Q^T Q = I; the (i,i)-th entry is 1; expanding this tells us that the i-th column has norm 1.
(b) Consider the (i,j)-th entry of the equation Q^T Q = I; when *i,j* are unequal, the (i,j)-th entry is 0; expanding this means that the i-th column is perpendicular to the j-th column.
(c) $\begin{bmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos t \end{bmatrix}$

3.1 #5 (a) For instance, the subspace consisting only of scalar multiples of A.

(b) Yes, since I = A – B.

(c) Consider the subspace consisting only of the following matrices:
$\begin{bmatrix} 0 & x \\ 0 & 0 \end{bmatrix}$

3.1 #10 Only (a), (d), (e).

3.1 #15 (a) line (but it could be a plane)
(b) point (but it could be a line)
(c) Say x,y \in X \cap Y. Then: x+y is in X, and x+y is in Y, so x+y is in X \cap Y; similarly cx is in X and cx is in Y so cx is in X \cap Y.

3.1 #20 (a) b_2 = 2b_1, b_1 + b_3 = 0 (b) b_1 + b_3 = 0

3.1 #23 unless the extra column lies in the span of the columns of A.

The column space gets bigger in the following example:
$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The column space stays the same in this example: [1] → [1 2]

#24 A = [1], B = [0], AB = [0] (the column space of AB is zero, while the column space of A is 1 -dimensional)

Q9 4 x 4 permutation matrices <==> permutations of {1,2,3,4}
the permutation matrix will be symmetric <==> the permutation is an involution
there are 10 involutions:
1, (12), (13), (14), (23), (24), (34), (12)(34), (14)(23), (13)(24); the corresponding matrices are:

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}0&0&1&0\\0&1&0&0\\1&0&0&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix} \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}$$

Q10

$$\begin{bmatrix}1&2&4\\2&4&8\\4&8&16\end{bmatrix}$$