18.06 Spring 2013 – Problem Set 1

This problem set is due Thursday, February 14th, 2013 at 4pm (hand in to Room 2-255). The textbook problems are out of the 4th edition. Problems 1-8 are worth 8 points. Problems 9 and 10 are worth 18 points each.

1. Do Problem 32 from Section 2.2.
2. Do Problem 3 & Problem 7 from Section 2.3.
3. Do Problem 11 from Section 2.4.
4. Do Problem 33 from Section 2.4.
5. Do Problem 36 from Section 2.4.
6. Do Problem 8 from Section 2.5.
7. Do Problem 25 from Section 2.5.
8. Do Problem 40 from Section 2.5.
9. The 3×3 matrix A is given as the sum of two other 3×3 matrices B and C satisfying:
   - all rows of B are the same vector u
   - all columns of C are the same vector v.

   Show that A is not invertible. One possible approach is to explain why there is a nonzero vector x satisfying both $Bx = 0$ and $Cx = 0$, so that $Ax = (B + C)x = Bx + Cx = 0$ has a nonzero solution.
10. A matrix A is called **symmetric** when its rows are the same as its columns. If we denote the entry in the i-th row and j-th column in A as a_{ij}, this means that $a_{ij} = a_{ji}$. For example,
$$\begin{bmatrix} 1 & 4 & 5 & 8 \\ 4 & 2 & 3 & 6 \\ 5 & 3 & 7 & 2013 \\ 8 & 6 & 2013 & 0 \end{bmatrix}$$
    is a symmetric matrix (here, $a_{34} = a_{43} = 2013$).

    A is **tridiagonal** when all the entries of A except in the middle three diagonals are zero. This means that $a_{ij} = 0$ if $|i - j| > 1$. An example of a tridiagonal matrix is
$$\begin{bmatrix} 1 & 4 & 0 & 0 & 0 \\ 6 & 2 & 7 & 0 & 0 \\ 0 & \pi & 7 & 2013 & 0 \\ 0 & 0 & 7.5 & 4 & 9 \\ 0 & 0 & 0 & 11 & 15 \end{bmatrix}.$$

    Problem. Construct a 3×3 tridiagonal matrix A with pivots $3, 4$, and 5 so that performing elimination steps on A goes:

- subtract row 1 from row 2.
- subtract row 2 from row 3.