18.06 Quiz 3 Professor Strang May 10, 2013

Please PRINT your name ______________________________ **1.**

2.

Please Circle Your Recitation: **3.**

r1	T	10	36-156	Russell Hewett	r7	T	1	36-144	Vinoth Nandakumar
r2	T	11	36-153	Russell Hewett	r8	T	1	24-307	Aaron Potechin
r3	T	11	24-407	John Lesieutre	r9	T	2	24-307	Aaron Potechin
r4	T	12	36-153	Stephen Curran	r10	T	2	36-144	Vinoth Nandakumar
r5	T	12	24-407	John Lesieutre	r11	T	3	36-144	Jennifer Park
r6	T	1	36-153	Stephen Curran					

(1) **(40 pts)**

In all of this problem, the 3 by 3 matrix A has eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with independent eigenvectors x_1, x_2, x_3.

(a) What are the trace of A and the determinant of A?

(b) Suppose: $\lambda_1 = \lambda_2$. Choose the true statement from 1, 2, 3:

1. A can be diagonalized. Why?
2. A can not be diagonalized. Why?
3. I need more information to decide. Why?

(c) From the eigenvalues and eigenvectors, how could you find the matrix A? Give a formula for A and explain each part carefully.

(d) Suppose $\lambda_1 = 2$ and $\lambda_2 = 5$ and $x_1 = (1, 1, 1)$ and $x_2 = (1, -2, 1)$. Choose λ_3 and x_3 so that A is symmetric positive semidefinite but not positive definite.

(2) **(30 pts.)**

Suppose A has eigenvalues $1, \frac{1}{3}, \frac{1}{2}$ and its eigenvectors are the columns of S:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{with} \quad S^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

(a) What are the eigenvalues and eigenvectors of A^{-1}?

(b) What is the general solution (with 3 arbitrary constants c_1, c_2, c_3) to the differential equation $du/dt = Au$? Not enough to write e^{At}. Use the c's.

(c) Start with the vector $u = (1, 4, 3)$ from adding up the three eigenvectors: $u = x_1 + x_2 + x_3$. Think about the vector $v = A^k u$ for VERY large powers k. What is the limit of v as $k \to \infty$?

(3) **(30 pts.)**

(a) For a really large number N, will this matrix be positive definite? Show why or why not.

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 4 & N & 1 \\ 3 & 1 & 4 \end{bmatrix}.$$

(b) Suppose: A is positive definite symmetric
Q is orthogonal (same size as A)
B is $Q^T AQ = Q^{-1}AQ$

Show that: 1. B is also symmetric.
2. B is also positive definite.

(c) If the SVD of A is $U\Sigma V^T$, how do you find the orthogonal V and the diagonal Σ from the matrix A?