18.06 Quiz 2 Professor Strang April 10, 2013

Please PRINT your name ________________________________ **1.**

2.

Please Circle your Recitation: **3.**

r1	T	10	36-156	Russell Hewett	r7	T	1	36-144	Vinoth Nandakumar
r2	T	11	36-153	Russell Hewett	r8	T	1	24-307	Aaron Potechin
r3	T	11	24-407	John Lesieutre	r9	T	2	24-307	Aaron Potechin
r4	T	12	36-153	Stephen Curran	r10	T	2	36-144	Vinoth Nandakumar
r5	T	12	24-407	John Lesieutre	r11	T	3	36-144	Jennifer Park
r6	T	1	36-153	Stephen Curran					

(1) **(40 pts)**

(a) If P projects every vector b in $\mathbb{R}^5$ to the nearest point in the subspace spanned by $a_1 = (1, 0, 1, 0, 4)$ and $a_2 = (2, 0, 0, 0, 4)$, what is the rank of P and **why?**

(b) If these two vectors are the columns of the 5 by 2 matrix A, which of the four fundamental subspaces for A is the nullspace of P?

(c) By Gram-Schmidt find an orthonormal basis for the column space of A (spanned by a_1 and a_2).

(d) If P is any (symmetric) projection matrix, show that $Q = I - 2P$ is an orthogonal matrix.

(2) **(30 pts.)**

(a) Find the determinant of the matrix A

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix}.$$

(b) The absolute value of $\det A$ tells you the volume of a box in $\mathbb{R}^4$. Describe that box (2 points – describe a different box with the same volume).

(c) Suppose you remove row 3 and column 4 of an invertible 5 by 5 matrix A. If that reduced matrix is not invertible, what fact does that tell you about A^{-1}?

(3) **(30 pts.)** This 4 by 4 Hadmard matrix is an orthogonal matrix. Its columns are orthogonal unit vectors.

$$Q = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}$$

(a) What projection matrix P_4 (give numbers) will project every b in $\mathbb{R}^4$ onto the line through q_4?

(b) What projection matrix P_{123} will project every b in $\mathbb{R}^4$ onto the subspace spanned by q_1, q_2, and q_3? Remember that those columns are orthogonal.

(c) Suppose A is the 4 by 3 matrix whose columns are q_1, q_2, q_3. Find the least-squares solution $\widehat{x}$ to the four equations

$$Ax = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = b.$$

What is the error vector e?