18.06 Professor Strang Quiz 2 April 11th, 2012

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

r01	T 11	4-159	Ailsa Keating	`ailsa`
r02	T 11	36-153	Rune Haugseng	`haugseng`
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r04	T 12	36-153	Rune Haugseng	`haugseng`
r05	T 1	4-153	Dimiter Ostrev	`ostrev`
r06	T 1	4-159	Uhi Rinn Suh	`ursuh`
r07	T 1	66-144	Ailsa Keating	`ailsa`
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r09	T 2	4-153	Dimiter Ostrev	`ostrev`
r10	ESG		Gabrielle Stoy	`gstoy`

1 (40 pts.)

(a) Find the projection p of the vector b onto the plane of a_1 and a_2, when

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ 7 \\ 1 \\ -7 \end{bmatrix}.$$

(b) What projection matrix P will produce the projection $p = Pb$ for every vector b in $\mathbb{R}^4$?

(c) What is the determinant of $I - P$? Explain your answer.

(d) What are all nonzero eigenvectors of P with eigenvalue $\lambda = 1$?

How is the number of independent eigenvectors with $\lambda = 0$ of an $n \times n$ square matrix A connected to the rank of A?

(You could answer (c) and (d) even if you don't answer (b).)

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2 (30 pts.)

(a) Suppose the matrix A factors into $A = PLU$ with a permutation matrix P, and 1's on the diagonal of L (lower triangular) and pivots $d_1, \ldots, d_n$ on the diagonal of U (upper triangular).

What is the determinant of A? EXPLAIN WHAT RULES YOU ARE USING.

(b) Suppose the first row of a new matrix A consists of the numbers $1, 2, 3, 4$. Suppose the cofactors C_{ij} of that first row are the numbers $2, 2, 2, 2$.

(Cofactors already include the $\pm$ signs.)

Which entries of A^{-1} does this tell you and what are those entries?

(c) What is the determinant of the matrix $M(x)$? For which values of x is the determinant equal to zero?

$$M(x) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & x \\ 1 & 1 & 4 & x^2 \\ 1 & -1 & 8 & x^3 \end{bmatrix}.$$

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3 (30 pts.)

(a) Starting from independent vectors a_1 and a_2, use Gram-Schmidt to find formulas for two orthonormal vectors q_1 and q_2 (combinations of a_1 and a_2):

$$q_1 =$$

$$q_2 =$$

(b) The connection between the matrices $A = [a_1\ a_2]$ and $Q = [q_1\ q_2]$ is often written $A = QR$. From your answer to Part (a), what are the entries in this matrix R?

(c) The least squares solution $\widehat{x}$ to the equation $Ax = b$ comes from solving what equation? If $A = QR$ as above, show that $R\widehat{x} = Q^T b$.

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