18.06 Quiz 1 Professor Strang March 9, 2012

Please PRINT your name ______________________________ **1.**

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Please Circle your Recitation: **3.**

4.

r1	T	11	4-159	Ailsa Keating
r2	T	11	36-153	Rune Haugseng
r3	T	12	4-159	Jennifer Park
r4	T	12	36-153	Rune Haugseng
r5	T	1	4-153	Dimiter Ostrev
r6	T	1	4-159	Uhi Rinn Suh
r7	T	1	66-144	Ailsa Keating
r8	T	2	66-144	Niels Martin Moller
r9	T	2	4-153	Dimiter Ostrev
r10	ESG			Gabrielle Stoy

1. **(36 pts.)** Suppose the 4 by 4 matrix A (with 2 by 2 blocks) is already reduced to its rref form

$$A = \begin{bmatrix} I & 3I \\ 0 & 0 \end{bmatrix}.$$

(a) Find a basis for the column space $C(A)$.

(b) Describe all possible bases for $C(A)$.

(c) Find a basis (special solutions are good) for the nullspace $N(A)$.

(d) Find the complete solution x to the 4 by 4 system

$$Ax = \begin{bmatrix} 5 \\ 4 \\ 0 \\ 0 \end{bmatrix}.$$

2. **(16 pts.)** Suppose the matrix A is m by n of rank r, and the matrix B is M by N of rank R. Suppose the column space $C(A)$ is contained in (possibly equal to) the column space $C(B)$. (This means that every vector in $C(A)$ is also in $C(B)$.) What relations must hold between m and M, n and N, and r and R?

It might be good to write down an example of A and B where all the columns are different.

3. (a) **(16 pts.)** Suppose three matrices satisfy $AB = C$. If the columns of B are dependent, show that the columns of C are dependent.

   (b) **(12 pts.)** If A is 5 by 3 and B is 3 by 5, show using part (a) or otherwise that $AB = I$ is impossible.

4. **(20 pts.)** Apply row elimination to reduce this invertible matrix from A to I. Then write A^{-1} as a product of three (or more) simple matrices coming from that elimination. Multiply these matrices to find A^{-1}.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 4 & 0 & 1 \end{bmatrix}.$$