18.06 Spring 2012 – Problem Set 2

This problem set is due Thursday, February 23rd, 2012 at 4pm (hand in to Room 2-106). The textbook problems are out of the 4th edition. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary('filename')` will start a transcript session, `diary off` will end one.)

1. Do Problems 7 & 9 from Section 2.6.
2. Do Problem 13 & 23 from Section 2.6.
3. Do Problem 6 from Section 2.7.
4. Do Problem 22 from Section 2.7.
5. Do Problem 38 from Section 2.7.
6. Do Problems 17 from Section 3.1.
7. Do Problem 23 from Section 3.1.
8. Do Problems 30 & 31 from Section 3.1.
9. This problem is about the vector space of matrices for a fixed number of rows and columns.

   (a) Explain carefully why the set of all 7×11 matrices forms a vector space (What is $c\,A + d\,B$? Which matrix is the zero vector?). Describe the simplest list of matrices you can think of which, allowing arbitrary linear combinations, will yield *all* 7×11 matrices. There should be 77 different matrices in your answer.

   (b) How many real number-valued parameters would you use to (unambiguously) describe the vector space $S_{3\times3}$ of 3×3 symmetric matrices (e.g. the set of all 3×3 matrices A such that $A^T = A$)? Identify *all* vector subspaces of $S_{3\times3}$ (it may be convenient to refer to the parameters you've introduced).

   (c) The 2×2 matrices with equal row sums ($a + b$ and $c + d$ are the same number), and equal column sums ($a + c$ and $b + d$), is a vector space. Find two matrices so that all these matrices are linear combinations of those two.

10. The MATLAB command `A = double(rand(2,2) < 0.5)` gives a random 2×2 matrix where each entry is either 0 or 1 (with equal probabilities).

    (a) Make a plot of the distribution of the number of pivots of the row-reduced versions (in MATLAB, the command `rank(A)` gives this number) of these random matrices. Here's some sample code that you can copy-paste into MATLAB:

```
clear; N=1000; num_zeros=0; num_ones=0; num_twos=0;
for i = 1:N
 A = double(rand(2,2) < 0.5);
 if rank(A)==2
```

```
  num_twos = num_twos + 1; %Then add one to that counter!
 end
 if rank(A)==1
  num_ones = num_ones + 1;
 end
 if rank(A)==0
  num_zeros = num_zeros + 1;
 end
end
distrib = [num_zeros num_ones num_twos]/N
bar([0 1 2], distrib, 0.1)
```

(b) Compare this to the exact probabilities of each value for the pivot number. Compute these by writing down all 16 possibilities and counting pivots.

(c) Extend the code in (a) to work for 5×5 matrices, and again show a histogram plot.

(d) For the 2×2 examples, what do you think the probability of having 2 pivots would be, if we took each matrix entry distributed continuously (and uniformly) in the *interval* $[0, 1]$? (No need to compute - but explain why!)