

Your PRINTED name is: _____ 1.

Your recitation number or instructor is _____ 2.

3.

1. (30 points)

- (a) Find the matrix P that projects every vector b in R^3 onto the line in the direction of $a = (2, 1, 3)$.
- (b) What are the column space and nullspace of P ? Describe them geometrically and also *give a basis for each space*.
- (c) What are *all* the eigenvectors of P and their corresponding eigenvalues? (You can use the geometry of projections, not a messy calculation.) The diagonal entries of P add up to _____ .

2. (30 points)

- (a) $p = A\hat{x}$ is the vector in $C(A)$ nearest to a given vector b . If A has independent columns, what equation determines \hat{x} ? What are all the vectors perpendicular to the error $e = b - A\hat{x}$? What goes wrong if the columns of A are dependent?
- (b) Suppose $A = QR$ where Q has orthonormal columns and R is upper triangular invertible. Find \hat{x} and p in terms of Q and R and b (not A).
- (c) (Separate question) If q_1 and q_2 are any orthonormal vectors in R^5 , give a formula for the projection p of any vector b onto the plane spanned by q_1 and q_2 (*write p as a combination of q_1 and q_2*).

3. (40 points) This problem is about the n by n matrix A_n that has zeros on its main diagonal and all other entries equal to -1 . In MATLAB $A_n = \text{eye}(n) - \text{ones}(n)$.

(a) Find the determinant of A_n . Here is a suggested approach:

Start by adding all rows (except the last) to the last row, and then factoring out a constant. (You could check $n = 3$ to have a start on part b.)

(b) For any invertible matrix A , the $(1, 1)$ entry of A^{-1} is the ratio of _____ .

So the $(1, 1)$ entry of A_4^{-1} is _____ .

(c) Find *two orthogonal eigenvectors* with $A_3 x = x$. (So $\lambda = 1$ is a double eigenvalue.)

(d) What is the third eigenvalue of A_3 and a corresponding eigenvector?