

Your PRINTED name is: _____ 1.

Your recitation number or instructor is _____ 2.

3.

4.

1. Forward elimination changes $A\mathbf{x} = \mathbf{b}$ to a row reduced $R\mathbf{x} = \mathbf{d}$: the complete solution is

$$\mathbf{x} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

(a) **(14 points)** What is the 3 by 3 reduced row echelon matrix R and what is \mathbf{d} ?

(b) **(10 points)** If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects R and \mathbf{d} to the original A and \mathbf{b} ? Use this matrix to find A and \mathbf{b} .

2. Suppose A is the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}.$$

(a) **(16 points)** Find all special solutions to $Ax = 0$ and describe in words the whole nullspace of A .

(b) **(10 points)** Describe the column space of this particular matrix A . “All combinations of the four columns” is not a sufficient answer.

(c) **(10 points)** What is the reduced row echelon form $R^* = \text{rref}(B)$ when B is the 6 by 8 block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix} \text{ using the same } A?$$

3. (16 points) Circle the words that correctly complete the following sentence:

(a) Suppose a 3 by 5 matrix A has rank $r = 3$. Then the equation $Ax = b$

(always / sometimes but not always)

has (a unique solution / many solutions / no solution).

(b) What is the column space of A ? Describe the nullspace of A .

4. Suppose that A is the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 2 & 4 \end{bmatrix}.$$

(a) **(10 points)** Explain in words how knowing all solutions to $A\mathbf{x} = \mathbf{b}$ decides if a given vector \mathbf{b} is in the column space of A .

(b) **(14 points)** Is the vector $\mathbf{b} = \begin{bmatrix} 8 \\ 28 \\ 14 \end{bmatrix}$ in the column space of A ?