18.06 Professor Johnson Quiz 3 May 1, 2009

Your PRINTED name is: ____________________

Please circle your recitation:

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(R09)	T2	2-284	Roman Bezrukavnikov

Grading

1	
2	
3	
4	
Total:	

1 (20 pts.) For each part, give **as much information as possible** about the **eigenvalues** of the matrix A described in that part. (Each part describes a *different* matrix A. A may be complex.)

(a) The recurrence $\mathbf{u}_{k+1} = A\mathbf{u}_k$ has a solution where $\|\mathbf{u}_k\| \to 0$ as $k \to \infty$ for one initial vector $\mathbf{u}_0$, but also has a solution with $\|\mathbf{u}_k\| \to \infty$ as $k \to \infty$ for a *different* choice of the initial vector $\mathbf{u}_0$.

(b) The equation $(A^2 - 4I)\mathbf{x} = \mathbf{b}$ has no solution for some right-hand side $\mathbf{b}$.

(c) $A = e^{B^T B}$ for some real matrix B with full column rank.

(d) $A = B^T B$ for a 4×3 real matrix B, and the matrix BB^T has eigenvalues $\lambda = 3, 2, 1, 0$. (Hint: think about the SVD of B.)

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2 (20 pts.) You are given the matrix

$$A = \begin{pmatrix} 0.5 & 0.2 & 0.2 \\ 0.1 & 0.5 & 0.5 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}.$$

(i) What are the eigenvalues of A? [*Hint:* Very little calculation required! You should be able to see two eigenvalues by inspection of the form of A, and the third by an easy calculation. You *shouldn't* need to compute $\det(A - \lambda I)$ unless you really want to do it the hard way.]

(ii) The vector $\mathbf{u}(t)$ solves the system

$$\frac{d\mathbf{u}}{dt} = A\mathbf{u}$$

for some initial condition $\mathbf{u}(0)$. If you are told that $\mathbf{u}(t)$ approaches some constant vector as $t \to \infty$, give as much true information as possible regarding the initial condition $\mathbf{u}(0)$.

[*Note:* be sure you understand that this is *not the same thing* as solving the recurrence $\mathbf{u}_{k+1} = A\mathbf{u}_k$! Imagine how you would find $\mathbf{u}(t)$ if you knew what $\mathbf{u}(0)$ was.]

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3 (10 pts.) The 3×3 matrix A has three independent eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ with corresponding eigenvalues λ_1, λ_2, and λ_3 (that is, $A\mathbf{v}_i = \lambda_i \mathbf{v}_i$ for $i = 1, 2, 3$).

If

$$\mathbf{b} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$$

for some coefficients c_1, c_2, and c_3, then write (in terms of λ_i, c_i, and $\mathbf{v}_i$) a formula for the solution $\mathbf{x}$ of

$$A^2\mathbf{x} + 2A\mathbf{x} - 3I\mathbf{x} = \mathbf{b}$$

(you can assume that a solution exists for any $\mathbf{b}$).

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4 (15 pts.) A is a 3×3 real-symmetric matrix. Two of its eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = -1$ with eigenvectors $\mathbf{v}_1 = (1, 1, 1)$ and $\mathbf{v}_2 = (1, -1, 0)$, respectively. The third eigenvalue is $\lambda_3 = 0$.

(I) Give an eigenvector $\mathbf{v}_3$ for the eigenvalue λ_3. (*Hint:* what must be true of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?)

(II) Using your result from (I), write the matrix e^A as the product of three matrices, and explicitly give the three matrices. (You need not work out the arithmetic, but your answer should contain no matrix inverses or matrix exponentials. *If you find yourself doing a lot of arithmetic, you are forgetting a useful property of this matrix!*)

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