18.06 Professor Johnson Quiz 2 April 1, 2009

Your PRINTED name is: ____________________

Please circle your recitation:

(R01)	M2	2-314	Qian Lin
(R02)	M3	2-314	Qian Lin
(R03)	T11	2-251	Martina Balagovic
(R04)	T11	2-229	Inna Zakharevich
(R05)	T12	2-251	Martina Balagovic
(R06)	T12	2-090	Ben Harris
(R07)	T1	2-284	Roman Bezrukavnikov
(R08)	T1	2-310	Nick Rozenblyum
(R09)	T2	2-284	Roman Bezrukavnikov

Grading
1
2
3
Total:

1 (20 pts.) (a) If P is the projection matrix onto the *null* space of A, then $P\mathbf{y} - \mathbf{y}$, for any $\mathbf{y}$, is in the ____________________ space of A.

(b) If $A\mathbf{x} = \mathbf{b}$ has a solution $\mathbf{x}$, then the closest vector to $\mathbf{b}$ in $N(A^T)$ is ____________________ (best answer).

(c) If the *rows* of A (an $m \times n$ matrix) are independent, then the dimension of $N(A^TA)$ is __________.

(d) If a matrix U has orthonormal *rows*, then $I =$ __________ and the projection matrix onto the *row* space of U is __________. (Your answers should be the simplest expressions involving U and U^T only.)

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2 (30 pts.) The matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & -7 \\ 2 & 4 & 1 & -5 \\ 1 & 2 & 2 & -16 \end{pmatrix}$$

is converted to row-reduced echelon form by the usual row-elimination steps, resulting in the matrix:

$$R = \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(♣) The *minimum* number of columns of A that form a *dependent* set of vectors is __________. The *maximum* number of columns of A that forms an *independent* set of vectors is __________.

(♢) Give an *orthonormal* basis for the *row* space of A. (Careful: be sure you start with a basis for the row space, not containing any dependent vectors.) Your answer may contain square roots left as $\sqrt{\textit{some number}}$.

(♠) Given the vector $\mathbf{b} = \begin{pmatrix} 2 & 5 & -9 & 3 \end{pmatrix}^T$, compute the *closest* vector $\mathbf{p}$ to $\mathbf{b}$ in the *row space* $C(A^T)$? (Hint: less calculation is needed if you use your answer from ♢.)

(♡) In terms of your answer $\mathbf{p}$ to ♠ above, what is the closest vector to $\mathbf{b}$ in the *nullspace* $N(A)$? (No calculation required, and you need not have solved ♠: you can leave your answer in terms of $\mathbf{p}$ and $\mathbf{b}$.)

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3 (20 pts.) You are told that the least-square linear fit to three points $(0, b_1)$, $(1, b_2)$, and $(2, b_3)$ is $C + Dt$ for $C = 1$ and $D = -2$. That is, the fit is $1 - 2t$.

In this question, you will work backwards from this fit to reason about the unknown values $\mathbf{b} = \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}^T$ at the coordinates $t = 0, 1, 2$.

(i) Write down the explicit equations that $\mathbf{b}$ must satisfy for $1 - 2t$ to be the least-square linear fit. (The points do *not* have to fall exactly on the line.)

(ii) If all the points fall *exactly* on the line $1 - 2t$, then $\mathbf{b} =$ ____________.
Check that this satisfies your equations in (i).

(iii) More generally, if all the points fall exactly on *any* straight line, then $\mathbf{b}$ is in the ________________ space of what matrix? (Write down the matrix.)

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