18.06 Professor Johnson Quiz 1 March 2, 2009

Your PRINTED name is: ____________________

Please circle your recitation:

(R01)	M2	2-314	Qian Lin
(R02)	M3	2-314	Qian Lin
(R03)	T11	2-251	Martina Balagovic
(R04)	T11	2-229	Inna Zakharevich
(R05)	T12	2-251	Martina Balagovic
(R06)	T12	2-090	Ben Harris
(R07)	T1	2-284	Roman Bezrukavnikov
(R08)	T1	2-310	Nick Rozenblyum
(R09)	T2	2-284	Roman Bezrukavnikov

Grading	
1	
2	
3	
Total:	

1 (20 pts.) Your classmate, Nyarlathotep, performed the usual elimination steps to convert A to echelon form U, obtaining:

$$U = \begin{pmatrix} 1 & 4 & -1 & 3 \\ 0 & 2 & 2 & -6 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Find a set of vectors spanning the nullspace $N(A)$.

(b) If $U\mathbf{y} = \begin{pmatrix} 9 \\ -12 \\ 0 \end{pmatrix}$, find the complete solution $\mathbf{y}$ (i.e. describe all possible solutions $\mathbf{y}$).

(c) Nyarla gave you a matrix

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{pmatrix}$$

and told you that $A = LU$. Describe the complete sequence of elimination steps that Nyarla performed, assuming that she did elimination in the usual way starting with the first column and eliminating downwards. That is, Nyarla first subtracted ______ times the first row from the second row, then subtracted ______ times the first row from the third row, then subtracted ____. (Be careful about signs: *adding* a multiple of a row is the same as subtracting a *negative* multiple of that row.)

(d) If $A\mathbf{x} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$, then $U\mathbf{x} =$ ______.

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2 (20 pts.) Which of the following (if any) are subspaces? For any that are *not* a subspace, give an example of how they violate a property of subspaces.

(I) Given some 3×5 matrix A with full row rank, the set of all solutions to $A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(II) All vectors $\mathbf{x}$ with $\mathbf{x}^T\mathbf{y} = 0$ *and* $\mathbf{x}^T\mathbf{z} = 0$ for some given vectors $\mathbf{y}$ and $\mathbf{z}$.

(III) All 3×5 matrices with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in their column space.

(IV) All 5×3 matrices with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in their nullspace.

(V) All vectors $\mathbf{x}$ with $\|\mathbf{x} - \mathbf{y}\| = \|\mathbf{y}\|$ for some given fixed vector $\mathbf{y} \neq 0$.

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3 (20 pts.) A is a matrix with a nullspace $N(A)$ spanned by the following three vectors:

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ -1 \\ 3 \\ 1 \end{pmatrix}.$$

(α) Give a matrix B such that its column space $C(B)$ is the same as $N(A)$. (There is more than one correct answer.) [Thus, any vector $\mathbf{y}$ in the nullspace of A satisfies $B\mathbf{u} = \mathbf{y}$ for some $\mathbf{u}$.]

(β) Give a different possible answer to (α): another B with $C(B) = N(A)$.

(γ) For some vector $\mathbf{b}$, you are told that a particular solution to $A\mathbf{x} = \mathbf{b}$ is

$$\mathbf{x}_p = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

Now, your classmate Zarkon tells you that a second solution is:

$$\mathbf{x}_Z = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix},$$

while your other classmate Hastur tells you "No, Zarkon's solution can't be right, but here's a second solution that is correct:"

$$\mathbf{x}_H = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 1 \end{pmatrix}.$$

Is Zarkon's solution correct, or Hastur's solution, or are both correct? (Hint: what should be true of $\mathbf{x} - \mathbf{x}_p$ if $\mathbf{x}$ is a valid solution?)

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