

18.06 Problem Set 3

Due Wednesday, 27 February 2008 at 4 pm in 2-106.

Problem 1: Do problem 7 from section 2.7 (pg. 105) in the book.

Problem 2: Do problem 10 from section 3.1 (pg. 119). (Give explanations of just a sentence or two.)

Problem 3: Consider the system of equations

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 5 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- a) For which right sides (find a condition on b_1, b_2, b_3) is this system solvable?
- b) Call the coefficient matrix A . Is the vector $(2, 5, -2)$ in the column space of A ? How about $(1, 2, 3)$?
- c) Suppose we add a fourth column $(2, 5, -2)$ to A . How does the column space change? What if we added the column $(1, 2, 3)$ instead?

Problem 4: Do problem 9 from section 3.2 (pg. 131).

Problem 5: Do problem 20 from section 3.2 (pg. 132).

Problem 6: Do problem 21 from section 3.2 (pg. 132). (Make sure the matrix you construct has exactly the nullspace asked for in the problem, and no larger.)

Problem 7: a) Do problem 1 from section 3.3 (pg. 141).

b) Do problem 3 in section 3.3 (pg. 141).

Problem 8: Do problem 17 in section 3.3 (pg. 143).

Problem 9: Define the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- a) Reduce A to ordinary echelon form. What are the pivots? What are the free variables?
- b) Find a special solution for each free variable. (Set the free variable to 1. Set the other variables to 0.)
- c) By combining the special solutions, describe every solution to $Ax = 0$.
- d) What is the rank of A ? Which columns will generate the column space $C(A)$?

Problem 10: Do problem 4 in section 3.4 (pg. 152).

Problem 11: Do problem 21 in section 3.4 (pg. 154).