

## 18.06 Problem Set 6

Due Wednesday, April 11, 2007 at 4:00 p.m. in 2-106

### **Problem 1** *Wednesday 4/4*

Do problem 9 of section 6.1 in your book.

### **Problem 2** *Wednesday 4/4*

Do problem 28 of section 6.1 in your book.

### **Problem 3** *Wednesday 4/4*

Do problem 33 of section 6.1 in your book.

### **Problem 4** *Wednesday 4/4*

Let  $A$  be a fixed  $n \times n$  matrix. We would like to find a matrix  $B$  such that  $AB = BA$ . This is the same as solving  $AB - BA = \text{zero matrix}$ . It turns out that this is a system of  $n^2$  equations on the entries of  $B$  (which are unknown). Since all these equations are linear, we can associate this system to a matrix  $M$ . Find an eigenvector of this matrix  $M$  with its corresponding eigenvalue.

### **Problem 5** *Monday 4/9*

Do problem 7 of section 6.2 in your book.

### **Problem 6** *Monday 4/9*

Do problem 10 of section 6.2 in your book.

### **Problem 7** *Monday 4/9*

Do problems 15 and 16 of section 6.2 in your book.

### **Problem 8** *Monday 4/9*

Do problem 22 of section 6.2 in your book.

### **Problem 9** *Monday 4/9*

Do problem 28 of section 6.2 in your book.

### **Problem 10** *Monday 4/9*

(a) Give an example of a  $3 \times 3$  matrix  $A \neq 0$  such that  $A^2 \neq 0$  but  $A^3 = 0$ . For your  $A$  find all the eigenvalues and the eigenvectors.

(b) Now, let  $B$  be a diagonalizable matrix such that there exists some positive integer  $k$  such that  $B^k = 0$ . Prove that  $B = 0$ .

(c) Does part (b) contradict part (a)? Explain your answer.