

## 18.06 Problem Set 5

Due Wednesday, March 21, 2007 at 4:00 p.m. in 2-106

### Problem 1 Wednesday 3/14

Do problem 10 of section 4.3 in your book.

### Problem 2 Wednesday 3/14

Do problem 17 of section 4.3 in your book.

### Problem 3 Wednesday 3/14

Find a function of the form  $f(t) = C\sin(t) + D\cos(t)$  that approximates the three points  $(0, 0)$ ,  $(\pi/2, 2)$  and  $(\pi, 1)$ . In other words, find coefficients  $C$  and  $D$  such that the error  $|f(0) - 0|^2 + |f(\pi/2) - 2|^2 + |f(\pi) - 1|^2$  is as small as possible.

### Problem 4 Wednesday 3/14

The MATLAB command `a=ones(n,1)` produces an  $n$ -by-1 matrix of ones. The command `r=(1:n)'` gives the vector  $(1, 2, \dots, n)$  transposed to a column by '. The command `s=r.^3` gives the column vector  $(1^3, 2^3, \dots, n^3)$ , because the dots means "a component at a time."

The purpose of this problem is to find the line  $y = c + dt$  closest to the cubic function  $y = t^3$  on the interval  $t = 0$  to  $t = 1$ .

(a) Find the best line using calculus, not MATLAB. Choose  $c$  and  $d$  to minimize

$$E(c, d) := \int_0^1 (c + dt - t^3)^2 dt$$

(Hint: find  $E(c, d)$  in terms of  $c$  and  $d$ , and use any method learned in 18.02 to minimize this.)

(b) With  $n = 15$ , choose  $C$  and  $D$  to give the line  $y = C + Dt$  that is closest to  $t^3$  at the points  $t = \frac{1}{15}, \frac{2}{15}, \dots, 1$ . Use MATLAB to do least squares in order to find  $C$  and  $D$ , and the differences  $c - C$  and  $d - D$ .

(Hint: Set up the equations you want to solve. Your equations (in matrix form) should involve the vectors  $\mathbf{a}$ ,  $\mathbf{r}/n$  and  $\mathbf{s}/n^3$ ).

(c) Repeat for  $n = 30$ . (Notice how  $\mathbf{r}/n$  and  $\mathbf{s}/n^3$  end at 1). Are the differences  $c - C$  and  $d - D$  smaller for  $n = 30$ ? By what factor?

### Problem 5 Friday 3/16

Consider in  $\mathbb{R}^4$  the subspace given by  $F = \{(x, y, z, w) : -x + y + 2z - w = 0\}$ .

(a) Give a basis for  $F$ .

(b) Use Gram-Schmidt to transform your basis into an orthonormal basis.

(c) What is the distance between the point  $(1, 3, 1, 1)$  and (the closest point to)  $F$ ?

**Problem 6** *Friday 3/16*

Do problem 6 of section 4.4 in your book.

**Problem 7** *Friday 3/16*

(a) Find a 3-by-3 orthogonal matrix  $A$  such that

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ and } A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

(b) How many matrices  $A$  are there that satisfy these conditions?

**Problem 8** *Monday 3/19*

Do problem 9 of section 5.1 in your book.

In the following problems explain how you calculated the determinants.

**Problem 9** *Monday 3/19*

Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}.$$

**Problem 10** *Monday 3/19*

(a) Calculate the determinants of the following "almost upper-triangular" matrices:

$$A_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

(b) Can you figure out how to continue the sequence of matrices  $A_2, A_3, A_4, A_5, \dots$  and calculate  $\det(A_n)$  for any  $n$ ?