18.06 Problem Set 3

Due Wednesday, Feb. 28, 2007 at **4:00 p.m.** in 2-106

Each problem is worth 10 points. The date next to the problem number indicates the lecture in which the material is covered.

Problem 1 *Tuesday 2/20*

For each of the following questions, please explain your answer.

(a) Let $F = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \geq y \geq z \geq 0 \}$. Is F a subspace of $\mathbb{R}^3$?

(b) The set of all real functions forms a vector space. Is the set of all functions of the form $f(x) = ax^2$ (where a can take any real value) a subspace? How about the set of all functions of the form $f(x) = x^2 + bx + c$ (where b and c can take any real value)?

(c) Let A be a fixed 3×2 matrix. Let F be the set of all 3×3 matrices B such that $BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Is F a subspace of the space of all 3×3 matrices?

Problem 2 *Tuesday 2/20*

Do problem 22 of section 3.1 in your book.

Problem 3 *Wednesday 2/21*

Do problems 5, 6 and 7 of section 3.2 in your book.

Problem 4 *Wednesday 2/21*

Do problem 25 of section 3.2 in your book.

Problem 5 *Friday 2/23*

Do problem 7 of section 3.4 in your book.

Problem 6 *Friday 2/23*

Do problem 21 of section 3.4 in your book.

Problem 7 *Friday 2/23*

Let $A = \begin{bmatrix} 1 & 2 & -2 & 1 & 0 \\ 2 & 4 & -3 & 3 & 0 \\ -3 & -6 & 5 & 4 & 2 \\ 5 & 10 & -9 & 6 & 0 \end{bmatrix}$.

(a) Transform A to (ordinary) echelon form.
(b) What are the pivots? What are the free variables?
(c) Now transform A to row reduced echelon form.
(d) Give the special solutions. What is the nullspace $N(A)$?
(e) What is the rank of A?

(f) Give the complete solution to $Ax = b$, where $b = A\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

Problem 8 *Monday 2/26*

Suppose the $m \times n$ matrix R is in row reduced echelon form $\begin{matrix} I & F \\ \mathbf{0} & \mathbf{0} \end{matrix}$, with r nonzero rows and first r columns as pivot columns.
(a) Describe the column space and the nullspace of R.
(b) Do the same for the $m \times 2n$ matrix $B = [R \quad R]$.
(c) Do the same for the $2m \times n$ matrix $C = \begin{matrix} R \\ R \end{matrix}$.
(d) Do the same for the $2m \times 2n$ matrix $D = \begin{matrix} R & R \\ R & R \end{matrix}$.

Problem 9 *Monday 2/26*

Do problem 17 of section 3.3 in your book.

Problem 10 *Monday 2/26*

Suppose the $m \times n$ matrix A ($m < n$) has a *right inverse* B, that is, a matrix B such that $AB = I$, the identity.
(a) What must the dimensions (the height and width) of B and of I be?
(b) Try calculating B in `MATLAB`: let $A = \begin{bmatrix} 4 & -3 & 1 \\ -2 & 0 & 2 \end{bmatrix}$ and use `A\I`. (This is the code in `MATLAB` for finding a matrix B such that $AB = I$. The $k \times k$ identity matrix is `eye(k)` in `MATLAB`.)
(c) Now try calculating B another way, with `rref([A I])`. (This is the reduced row echelon form, the result of Gauss-Jordan elimination.) What do you get? Use your result to state another, different, B with $AB = I$. Why is B not unique?
(d) Why can't there be a left inverse $CA = I$?