18.06 Problem Set 1

Due Wednesday, Feb. 14, 2007 at **4:00 p.m.** in 2-106

Each problem is worth 10 points. The date next to the problem number indicates the lecture in which the material is covered.

Problem 1 *Wednesday 2/07*

Consider the following system of equations:

$$\begin{aligned} x + 3y + 2z &= 6 \\ 2x + 5y + 4z &= 1 \\ 3x + 8y + 6z &= 7 \end{aligned}$$

What do you notice about the equations?
The first two planes intersect in a line. What do you know about that line and the third plane? How many solutions does the system have?

Problem 2 *Wednesday 2/07*

(a) Find a matrix A such that $A\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$ and $A\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

(b) What is $A\begin{bmatrix} 3 \\ 3 \end{bmatrix}$?

Problem 3 *Wednesday 2/07*

Do problem 26 of section 2.1 in your book.

Problem 4 *Wednesday 2/07*

Let's practice using Matlab to check that in general AB and BA are not equal. (Hint: you can type `diary` at the beginning of your session to save a transcript.)
Let's start with matices of different sizes. Let `A=ones(3,2)` and `B=ones(2,3)` (that is, the 3-by-2 and 2-by-3 matrices with all entries equal to 1). Compute AB and BA. What are their sizes?
Now, let's multiply to 3-by-3 matrices. Let `C=[a b c; d e f; g h i]`, where `a...i` are nine of your favorite numbers. *Now let the computer pick one:* `D=rand(3,3)` gives us a random 3-by-3 matrix. What are CD and DC? Are they equal?

Problem 5 *Friday 2/09*

Write examples of systems $A\vec{x} = \vec{b}$ where A is a 3-by-3 matrix and:

1. the three planes meet in a common line
2. in the row picture, all three planes are parallel but distinct
3. the intersection of the first two planes does not intersect the third plane
4. $\vec{b}$ is not a linear combination of the columns of A.
5. in the column picture, $\vec{b}$ is a multiple of the second column of A.

Problem 6 *Friday 2/09*

Answer the following questions for the systems in problem 5:
(a) How many solutions does each have? Describe the shape (point, line, ...) of each solution set.
(b) Reduce each by elimination (you need not back-substitute) and check your answer.

Problem 7 *Friday 2/09*

Solve the following system by elimination and back substitution:

$$\begin{aligned} 2x+3y+z &= 0 \\ x-2y-z &= -3 \\ x+y+2z &= 3 \end{aligned}$$

Write down the elimination matrices E_{21}, E_{31}, E_{32} you used.

Problem 8 *Monday 2/12*

Consider the matrices $A=\begin{bmatrix} 5 & -3 & -9 \\ 2 & 4 & -1 \\ -1 & 7 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -1 \\ 0 & 2 \\ -1 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ -3 & 3 \end{bmatrix}$.
(a) Find AB and AC.
(b) Do you notice anything special? Why does this tell you A is not invertible?

Problem 9 *Monday 2/12*

Do problem 13 of section 2.4 in your book.

Problem 10 *Monday 2/12*

Do problem 7 of section 2.5 in your book.