18.06 Spring 2006 - Exam 2 Review Problems

SOLUTIONS TO SELECTED PROBLEMS

1. Section 4.1, Problem 19

Answer: Suppose $\mathbf{L}$ is a one-dimensional subspace in $\mathbb{R}^3$. It's orthogonal complement $\mathbf{L}^\perp$ is the **2-dimensional subspace (a plane)** perpendicular to $\mathbf{L}$. The $(\mathbf{L}^\perp)^\perp$ is a **1-dimensional subspace (a line)** perpendicular to $\mathbf{L}^\perp$. In fact $(\mathbf{L}^\perp)^\perp$ is the same as $\mathbf{L}$.

2. Section 4.1, Problem 20

Answer: Suppose $\mathbf{V}$ is the whole space $\mathbb{R}^4$. Then $\mathbf{V}^\perp$ contains only the vector **the zero vector**. Then $(\mathbf{V}^\perp)^\perp$ is $\mathbf{V} = \mathbb{R}^4$.

3. Section 4.1, Problem 27

Answer: The lines $3x + y = b_1$ and $6x + 2y = b_2$ are **parallel**. They are the same line if $\mathbf{2b_1 = b_2}$. In that case (b_1, b_2) is perpendicular to the vector $(\mathbf{2}, -\mathbf{1})$. The nullspace of the matrix is the line $\mathbf{3x + y = 0}$. One particular vector in that nullspace is $(-\mathbf{1}, \mathbf{3})$.

4. Section 4.2, Problem 14

Answer: The projection of $\mathbf{b}$ onto the column space of A is $\mathbf{b}$ itself, but P is not necessarily I.

$$P = \frac{1}{21}\begin{bmatrix} 5 & 8 & -4 \\ 8 & 17 & 2 \\ -4 & 2 & 20 \end{bmatrix} \text{ and } p = (0, 2, 4).$$

5. Section 4.2, Problem 25

Answer: The column space of P will be S (n-dimensional). Then $r =$ dimension of the column space$= n$.

6. Section 4.3, Problem 19

Answer: $\hat{\mathbf{x}} = (0, 0)$. If $\mathbf{b} = \mathbf{e}$, then $\mathbf{b}$ is perpendicular to the column space of A. The projection $\mathbf{p} = 0$.

7. Section 4.3, Problem 20

Answer: $\hat{\mathbf{x}} = (9, 4)$ and $\mathbf{e} = (0, 0)$. The error $\mathbf{e} = (0, 0)$ because $\mathbf{b}$ is in the column space of A.

8. Section 4.4, Problem 4

Answer:

a) $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, $QQ^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $(1, 0)$ and $(0, 0)$ are orthogonal but not independent.

c) $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}), (\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}), (-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$

9. Section 4.4, Problem 5

Answer: Two orthogonal vectors are $(1, -1, 0)$ and $(1, 1, -1)$. Orthonormal vectors are $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)$ and $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$.

10. Section 5.1, Problem 2

Answer: If A is 3×3 and $\det(A) = -1$, then

$\det(\frac{1}{2}A) = (\frac{1}{2})^3\det(A) = -\frac{1}{8}$,

$\det(-A) = (-1)^3\det(A) = -1$,

$\det(A^2) =$det$(A)\cdot$det$(A) = 1$,

$\det(A^{-1}) = 1/\det(A) = -1$.

11. Section 5.1, Problem 29

Answer: If A is rectangular (not square), then $\det(A)$, $\det(A^T)$ are not defined.

12. Section 5.3, Problem 15

Answer:

a) Cofactors C_{21}, C_{31}, C_{32} are all zero.

b) $C_{12} = C_{21}, C_{31} = C_{13}, C_{32} = C23$.