18.06 Spring 2006 - Problem Set 6

SOLUTIONS TO SELECTED PROBLEMS

1. Section 5.1, Problem 8

Answer: There are $5! = 120$ permutation matrices. $5!/2 = 60$ have det$= +1$.

A permutation matrix that needs four exchanges to reach the identity matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Section 5.2, Problem 25

Answer:

a) If we use the big formula to find the determinant, picking an entry from B requires picking an entry from the zero block which results in zero. This leaves a pair of entries from A times a pair from D leading to $(detA)(detD)$.

b) and c)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Section 5.3, Problem 1

Answer:

a) $\det A = 3$, $\det B_1 = -6$, $\det B_2 = 3$. Therefore $x_1 = -6/3 = -2$ and $x_2 = 3/3 = 1$.

b) $\det A = 4$, $\det B_1 = 3$, $\det B_2 = -2$, $\det B_3 = 1$. Therefore $x_1 = 3/4$, $x_2 = -1/2$, $x_3 = 1/4$.

4. Section 5.3, Problem 7

Answer: If all the cofactors are 0, then $\det A = 0$ (by the Cofactor Formula for determinants) and A has no inverse.

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ has no zero cofactors but it is not invertible.