18.06 Spring 2006 - Problem Set 2

SOLUTIONS TO SELECTED PROBLEMS

1. Section 2.6, Problem 13

Answer: $A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a & a \\ & b-a & b-a & b-a \\ & & c-b & c-b \\ & & & d-c \end{bmatrix}$

Need $a \neq 0$, $b \neq a$, $c \neq b$, $d \neq c$.

2. Section 2.6, Problem 16

Answer: $L\mathbf{c} = \mathbf{b} : \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{c} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ gives $\mathbf{c} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$.

$U\mathbf{x} = \mathbf{c} : \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ gives $\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

$A = LU = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

3. Section 2.6, Problem 28

Answer: $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & c & 1 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & c-6 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

$c = 6$ and $c = 7$ makes LU impossible because $c = 6$ needs a row exchange and $c = 7$ will have only 2 pivots.

4. Section 2.7, Problem 7

Answer:

a) False

b) False

c) True

d) True

5. Section 2.7, Problem 11

Answer: $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$; $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Multiplying A on the right by P_2 exchanges the **columns** of A.

6. Section 2.7, Problem 13

Answer: $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ or its transpose have $P^3 = I$.

$\widehat{P} = \begin{bmatrix} 1 & 0 \\ 0 & P \end{bmatrix}$ for the previous P has $\widehat{P}^4 \neq I$.

7. Section 2.7, Problem 17

Answer:

a) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ has $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

8. Section 2.7, Problem 19

Answer:

a) $(R^TAR)^T = R^T(R^TA)^T = R^TA^T(R^T)^T = R^TAR$. R^TAR is $n \times n$.

b) $(R^TR)_{jj}$ = (column j of R) $\cdot$ (column j of R) = length squared of column j.

9. Section 3.1, Problem 10

Answer:

(a), (d), (e) are the only ones that are subspaces.

10. Section 3.1, Problem 19

Answer:

The column space of A is the $x-$ axis = all vectors of the form $(x, 0, 0)$.

The column space of B is the xy-plane = all vectors of the form $(x, y, 0)$.

The column space of C is the line vectors of the form $(x, 2x, 0)$.

11. Section 3.1, Problem 27

*A*nswer:

a) False; $\mathbf{0} \in \mathbf{C}(A)$, hence the set of vectors not in $\mathbf{C}(A)$ will not contain $\mathbf{0}$ and is not a subspace.

b) True

c) True

d) False; if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $\mathbf{C}(A) = \mathbb{R}^2$ and $\mathbf{C}(A - I) = \mathbf{0}$.