18.06 Professor Edelman Quiz 1 March 6, 2006

Your PRINTED name is: SOLUTIONS

Grading

1

2

3

Please circle your recitation:

1)	M 2	2-131	A. Kasimov	2-339	3-1715	`kasimov`
2)	M 3	2-131	A. Kasimov	2-339	3-1715	`kasimov`
3)	M 3	2-132	R. Lehman	2-251	3-7566	`rclehman`
4)	T 10	2-132	F. Liu	2-333	3-7826	`fuliu`
5)	T 11	2-132	P. Shor	2-369	3-4362	`shor`
6)	T 12	2-132	P. Shor	2-369	3-4362	`shor`
7)	T 1	2-131	F. Liu	2-333	3-7826	`fuliu`
8)	T 1	2-132	A. Osorno	2-229	3-1589	`aosorno`
9)	T 2	2-132	A. Osorno	2-229	3-1589	`aosorno`

1 (30 pts.) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 2 & 4 \end{bmatrix}$$

(a) By eliminating, prove that the column space consists of those vectors $(x, y, z)^{\mathrm{T}}$ with $2x - y = 0$.

(b) By eliminating, prove that the row space consists of those vectors $(x, y, z)^{\mathrm{T}}$ with $y + z = 2x$.

Solution.

(a) $$\begin{bmatrix} 1 & 1 & 1 & x \\ 2 & 2 & 2 & y \\ 3 & 2 & 4 & z \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & x \\ 0 & 0 & 0 & y-2x \\ 0 & -1 & 1 & z-3x \end{bmatrix}$$

$$\Longrightarrow y - 2x = 0 \Longleftrightarrow 2x - y = 0.$$

(b) $CS(A^{\mathrm{T}}) = RS(A)$

$$\begin{bmatrix} 1 & 2 & 3 & x \\ 1 & 2 & 2 & y \\ 1 & 2 & 4 & z \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 & x \\ 0 & 0 & -1 & y-x \\ 0 & 0 & 1 & z-x \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 & x \\ 0 & 0 & -1 & y-x \\ 0 & 0 & 0 & z-x+y-x \end{bmatrix}$$

$$\Longrightarrow z + y - 2x = 0 \Longleftrightarrow z + y = 2x.$$

2 (40 pts.) For each of the eight cases below, exhibit a 3×3 matrix A with rank r and the specified condition or argue convincingly that it is impossible.

(a) $r = 0$, $\text{col}(A) = \text{row}(A)$

(b) $r = 1$, $\text{col}(A) = \text{row}(A)$

(c) $r = 2$, $\text{col}(A) = \text{row}(A)$

(d) $r = 3$, $\text{col}(A) = \text{row}(A)$

(e) $r = 0$, $\text{col}(A) \neq \text{row}(A)$

(f) $r = 1$, $\text{col}(A) \neq \text{row}(A)$

(g) $r = 2$, $\text{col}(A) \neq \text{row}(A)$

(h) $r = 3$, $\text{col}(A) \neq \text{row}(A)$

Solution.

(a) 3×3 0 matrix

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) 3×3 identity matrix

(e) Not possible.

$r = 0 \implies CS(A) = 0 = RS(A)$

(f) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(h) Not possible.

$r = 3 \implies CS(A) = \mathbb{R}^3 = RS(A).$

3 (30 pts.) Let

$$A = \begin{bmatrix} 1 & a & 0 & d & 0 & g \\ 0 & b & 1 & e & 0 & h \\ 0 & c & 0 & f & 1 & i \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}$$

(a) Find the complete solution to $Ax = v$, if $s = 1$.

(b) Find the complete solution to $Ax = v$, if $s = 0$.

(*Hint:* Best if you don't work too hard!)

Solution.

(a) No solution.

The last row of A is all zeros. If $s = 1$, we get $0 = 1$.

(b) Let $\overline{x} = (x_1, x_2, x_3, x_4, x_5, x_6)$ and let x_2, x_4, x_6 be free variables.

$$x_{\mathsf{particular}} = \begin{bmatrix} p \\ 0 \\ q \\ 0 \\ r \\ 0 \end{bmatrix}$$

To find the special solutions:

$$\text{let}\quad \begin{matrix} x_2=-1 \\ \\ x_4=\ \ 0 \\ \\ x_6=\ \ 0 \end{matrix} \quad \begin{bmatrix} a \\ -1 \\ b \\ 0 \\ c \\ 0 \end{bmatrix}, \qquad \begin{matrix} x_2=\ \ 0 \\ \\ x_4=-1 \\ \\ x_6=\ \ 0 \end{matrix} \quad \begin{bmatrix} d \\ 0 \\ e \\ -1 \\ f \\ 0 \end{bmatrix}, \qquad \begin{matrix} x_2=\ \ 0 \\ \\ x_4=\ \ 0 \\ \\ x_6=-1 \end{matrix} \quad \begin{bmatrix} g \\ 0 \\ h \\ 0 \\ i \\ -1 \end{bmatrix}$$

The complete solution:

$$\overline{x} = \begin{bmatrix} p \\ 0 \\ q \\ 0 \\ r \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} a \\ -1 \\ b \\ 0 \\ c \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} d \\ 0 \\ e \\ -1 \\ f \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} g \\ 0 \\ h \\ 0 \\ i \\ -1 \end{bmatrix}$$