

Your PRINTED name is: \_\_\_\_\_

**Grading**  
1  
2  
3

Please circle your recitation:

- 1) M 2 2-131 A. Kasimov 2-339 3-1715 kasimov
- 2) M 3 2-131 A. Kasimov 2-339 3-1715 kasimov
- 3) M 3 2-132 R. Lehman 2-251 3-7566 rclehman
- 4) T 10 2-132 F. Liu 2-333 3-7826 fuliu
- 5) T 11 2-132 P. Shor 2-369 3-4362 shor
- 6) T 12 2-132 P. Shor 2-369 3-4362 shor
- 7) T 1 2-131 F. Liu 2-333 3-7826 fuliu
- 8) T 1 2-132 A. Osorno 2-229 3-1589 aosorno
- 9) T 2 2-132 A. Osorno 2-229 3-1589 aosorno

1 (30 pts.) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 2 & 4 \end{bmatrix}$$

- (a) By eliminating, prove that the column space consists of those vectors  $(x, y, z)^T$  with  $2x - y = 0$ .
- (b) By eliminating, prove that the row space consists of those vectors  $(x, y, z)^T$  with  $y + z = 2x$ .

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**2 (40 pts.)** For each of the eight cases below, exhibit a  $3 \times 3$  matrix  $A$  with rank  $r$  and the specified condition or argue convincingly that it is impossible.

(a)  $r = 0, \text{col}(A) = \text{row}(A)$

(e)  $r = 0, \text{col}(A) \neq \text{row}(A)$

(b)  $r = 1, \text{col}(A) = \text{row}(A)$

(f)  $r = 1, \text{col}(A) \neq \text{row}(A)$

(c)  $r = 2, \text{col}(A) = \text{row}(A)$

(g)  $r = 2, \text{col}(A) \neq \text{row}(A)$

(d)  $r = 3, \text{col}(A) = \text{row}(A)$

(h)  $r = 3, \text{col}(A) \neq \text{row}(A)$

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**3 (30 pts.)** Let

$$A = \begin{bmatrix} 1 & a & 0 & d & 0 & g \\ 0 & b & 1 & e & 0 & h \\ 0 & c & 0 & f & 1 & i \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}$$

(a) Find the complete solution to  $Ax = v$ , if  $s = 1$ .

(b) Find the complete solution to  $Ax = v$ , if  $s = 0$ .

(*Hint:* Best if you don't work too hard!)

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