18.06 Professor A.J. de Jong Exam 2 April 9, 2003

**Your name is:** ______________________________

**Please circle your recitation:**

Important: Briefly explain all of your answers.

**1 (29 pts.)**

(a) Compute the determinant of the following matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 & 2 \end{pmatrix}$$

Mention the method used for each step in the calculation.

(b) Give a basis for each of the four fundamental subspaces associated to the following matrix

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

**2 (29 pts.)**

(a) Apply the Gram-schmidt algorithm to the columns of the matrix $A$ below. (Use the order in which they occur in the matrix!) Use this to write $A = QR$, where $Q$ is a matrix with orthonormal columns, and $R$ is upper triangular.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & -1 \end{pmatrix}.$$

(b) Compute the matrix of the projection onto the column space of $A$. What is the distance of the point $(1, 1, 1, 0)$ to this column space?

**3 (14 pts.)** Show that the following determinant is zero for any values of $a$, $b$ and $c$:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

**4 (28 pts.)** Let $A$ be the matrix

$$\begin{pmatrix} 7 & 5 \\ 3 & -7 \end{pmatrix}.$$

(a) Find matrices $S$ and $\Lambda$ such that $A$ has a factorization of the form

$$A = S\Lambda S^{-1},$$

where $S$ is invertible and $\Lambda$ is diagonal: $\Lambda = \text{diag}(\lambda_1, \lambda_2)$.

(b) Find a matrix $B$ such that $B^3 = A$. (Hint: First find such a matrix for $\Lambda$. Then use the formula above.)