18.06 Professor A.J. de Jong Exam 1 March 3, 2003

Your name is: ______________________________

Please circle your recitation:

1 (30 pts.)

(a) Compute the following matrix product

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ -5 & -4 & -3 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$$

No explanation is necessary.

(b) Let U be the matrix below. Reduce U to a reduced row echelon matrix by row operations (upward elimination). Find the "special solutions" to $Ux = 0$. Also give an expression for the general solution to $Ux = 0$.

$$U = \begin{pmatrix} 1 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 7 & 5 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$$

2 (35 pts.)

(a) Let A and b be as below. For any real number t, and any real number s: Find the complete solution to the equation $Ax = b$ using the algorithm described in class and in the book. (It depends on t and s.)

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & t \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 0 \\ 0 \\ s \end{pmatrix}$$

(b) First part: For which t are the columns of the matrix A linearly dependent? Second part: Consider b and the first three columns of A. For which s are these linearly dependent?

3 (35 pts.) The elimination algorithm explained in the course (with "row swapping after Gaussian elimination") was applied to the matrix A. Suppose it yields the following equality:

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 11 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 10 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

(a) Which row operations do the four elimination matrices in the product correspond to? Please write them down in words in the order in which they were performed on A. Why is the upper left hand corner of A zero? (This is the $(1,1)$ entry of A.)

(b) The equation implies that A factors as $A = LPUR$. Here R is the matrix on the right hand side of the $=$ sign. The matrices U, P, and L are invertible 4×4 matrices. The matrix U is upper triangular. The matrix P is a permutation matrix. And L is lower triangular. Find U, P, and L, and explain how you got them.