Problem 1 (a) After forming the augmented matrix and doing row reduction, the third row becomes $\begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}$, which corresponds to the equation 0 = -1, so there is no solution.

- (b) The same argument shows that in order for Ax = b to have a solution, b must satisfy $b_3 = b_1 + b_2$.
- (c) If A were invertible, there would always be a solution Ax = b.

Problem 2 (a)

$$A=LU=\left[egin{array}{ccc} 1 & 0 & 0 \ 1 & 1 & 0 \ 0 & 1 & 1 \end{array}
ight]\left[egin{array}{ccc} 2 & 2 & 1 \ 0 & 3 - 1 \ 0 & 0 & 3 \end{array}
ight].$$

(b) From U in part (a) we see that every column is a pivot column. The pivot columns from A are a basis for the column space: (2,2,0), (2,5,3), (1,0,2). Since the rank is three, the column space is all of \mathbf{R}^3 , so another basis would be the standard basis (1,0,0), (0,1,0), (0,0,1). In fact, any three independent vectors in \mathbf{R}^3 will do.

(c) The rank is three because there are three pivots.

Problem 3 (a) This is an LU factorization. The U is the echelon form of A, so you can see that there are three pivots, so the rank of A is three.

- (b) A basis for N(A) consists of the special solutions. These are (-1, -2, 1, 0, 0) and (-1, 1, 0, -1, 1).
- (c) A particular solution is (-30, -15, 0, 10, 0) so the complete solution is

$$egin{bmatrix} -30 \ -15 \ 0 \ 10 \ 0 \end{bmatrix} + x_3 egin{bmatrix} -1 \ -2 \ 1 \ 0 \ 0 \end{bmatrix} + x_5 egin{bmatrix} -1 \ 1 \ 0 \ -1 \ 1 \end{bmatrix}.$$

Problem 4 (a) One basis would be $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

- (b) The subspace consisting of all multiples of A is a subspace which contains A but not B.
- (c) True: If a subspace V contains A and B, then it contains A B = I.
- (d) Same answer as (b) will work.

Problem 5 There are many different proofs. One is to say that if $A^2 = 0$ then obviously A^2 is not invertible. Therefore A isn't invertible, because the product of invertible matrices is invertible. I.e., if A were invertible, then A^2 would be invertible.