18.06 Professor Strang Quiz 1 Solutions October 8, 1999

1 (a) $N(A) = N(B)$ and $C(A^T) = C(B^T)$

(b) $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix}$ for the row space; $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ -5 \\ 1 \end{bmatrix}$ for the nullspace.

(c) **True**
Reason: Whenever a combination $cx + dy = 0$, multiply by A to see that $c(Ax) + d(Ay) = 0$.

2 (a) $\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ (The first matrix is invertible so it has no effect on the nullspace)

(b) The pivot columns are 1, 2, 4 (and the first matrix has an effect!) $\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 28 \end{bmatrix}$.

(c) $$x = x_p + x_n = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$

3 (a) Those vectors y are dependent, they span a space $N(A^T)$ that has dimension 2. So $m - r = 2$ and $m = 3$ and $r = 1$.

(b) The second block of rows copies the first so no increase in the rank. Same for the second block of columns. So those extra blocks leave the rank unchanged.

(c) If $r = m$ then $Ax = b$ has a solution (one or more) for *every* right side b.

4 (a)–(b) The particular solution says that column 2 + column 3 = right side b. The nullspace solution says that 2(column 2) + column 3 = 0.
Therefore column 2 = $-b$ and column 3 = $2b$.

(c) Since the nullspace is one-dimensional, the 3 by 4 matrix A has rank 2. Therefore we know that the first column of A is *not* a multiple of b.