3 December 1997 **Profs. S. Lee and A. Kirillov**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam III for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Show all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results.
Point values (total of 100) are marked on the left margin.

1. Let $A = \begin{bmatrix} 4 & 1 \\ -1 & 4 \end{bmatrix}$.

[10] **1a.** Find the eigenvalues of A.

[10] **1b.** Find an eigenvector for each eigenvalue of A.

[10] **1c.** Compute $x_1^H x_2$.
(Note: x_1 and x_2 are the complex eigenvectors that you obtained in **1b**.)

2. Let $A = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$.

[12] **2a.** Find an invertible matrix S that makes $S^{-1}AS$ a diagonal matrix.

[10] **2b.** For the differential equation $\frac{du}{dt} = Au$, give a nonzero initial vector $u(0)$ such that $u(t) \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as $t \to \infty$.

[16] **3.** Fill in the matrix $A = \begin{bmatrix} 0.5 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ so that A is a positive Markov matrix with the steady state vector $x_1 = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$.

(Recall that the limit of $A^k u_0$ is always a multiple of x_1.)

4. Each independent question refers to the matrix $A = \begin{bmatrix} 4 & 1 \\ d & -4 \end{bmatrix}$.

In each case, find the value of d that makes the statement true (and show your work!).

[10] **4a.** Give a value for d such that $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ is an eigenvector of A.

[10] **4b.** Give a value for d such that 2 is one of the eigenvalues of A.

[12] **4c.** Give a value for d such that A is a nondiagonalizable matrix.

Recall that $A = \begin{bmatrix} 4 & 1 \\ d & -4 \end{bmatrix}$.