Second Midterm Review Problems

Problem 1: Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -2 & 0 \end{bmatrix}.$$

and the vector

$$b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(1) Find $p = Ax$ that minimizes $\|Ax - b\|$.

(2) Find x that minimizes $\|Ax - b\|$.

Problem 2: Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ -1 & -3 \\ -1 & -3 \end{bmatrix}$$

(1) Use Gram-Schmidt to find the factorization $A = QR$.

(2) Check that the matrix in (1) satisfies $Q^T Q = I$

Problem 3: Consider the linear transformation

$$\phi : \mathbb{R}^3 \to \mathbb{R}^2$$

such that:

- $\phi(e_1) = 3e_1 + e_2$
- $\phi(e_2) = 2e_1$
- $\phi(e_3) = e_1 + e_2$

Here recall that we denote by e_i the standard basis.

(1) Find the matrix A of ϕ with respect to the standard basis.

(2) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ and let $w_1 = \phi(v_1)$ and $w_2 = \phi(v_2)$. What is the matrix B of ϕ with respect to the bases $\{v_i\}$ and $\{w_j\}$.

Problem 4: Consider the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(1) Find the determinant using the cofactor formula along the first row.

(2) Find the determinant using the cofactor formula along the second row.

(3) Use Cramer's rule to find the inverse of the above matrix.

Problem 5: Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 5 & 7 \end{bmatrix}$$

(1) Use the cofactor formula to compute the determinant.

(2) Use row operations to compute the determinant.

(3) Use the large 3! formula to compute the determinant.

Problem 6 Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$$

(1) Compute the eigenvalues of the matrix.

(2) Compute eigenvectors for the above eigenvalues. Is there an eigenvector that is particularly easy?