First Midterm Review Problems

Problem 1: Consider the matrix

$$A = \begin{bmatrix} 3 & 6 & 3 & 9 \\ -2 & -4 & 0 & 2 \end{bmatrix}.$$

(1) Compute the reduced row echelon form of A.

(2) What is a basis for the column space of A? What is a basis for the nullspace of A? What is the rank of A?

(3) Write down a general solution to the following system of 2 equations in 4 variables:

$$3a + 6b + 3c + 9d = 0$$

$$-2a - 4b + 2d = -6.$$

(4) Write down a basis for the column space of A^T, and for the nullspace of A^T.

Problem 2: Write down the 4×4 elimination matrix $E_{3,1}^{(-2)}$. What is its inverse?

Problem 3: Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & 8 & 11 \\ -2 & -4 & -8 & -8 \\ 1 & 1 & 2 & 3 \end{bmatrix}.$$

(1) Compute the LU factorization of A. Recall that L is lower-triangular with 1s along the diagonal, and U is upper triangular. The matrix U is obtained by bringing A into row echelon form (but not into reduced row echelon form).

(2) Write L as a product of 4×4 elimination matrices.

(3) What is a basis for the column space of A? What is the rank of A? Is A invertible?

(4) Let A' denote the matrix

$$A' = \begin{bmatrix} -2 & -4 & -8 & -8 \\ 1 & 1 & 2 & 3 \\ 2 & 4 & 8 & 11 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$

Construct a $PA' = LU$ decomposition by finding P, L, and U such that P is a permutation matrix, L is lower triangular with 1s on the diagonal, and U is upper triangular.