Recitation 2. September 17

Focus: multiplying matrices (and taking inverses), $A = LU$, $A = LDU$ and $PA = LU$ factorizations, transposes, symmetric matrices

The LU factorization of a square matrix A is the unique way of writing it:

$$\boxed{A = LU}$$

where L is a lower-diagonal matrix with 1 on the diagonal and U is an upper-diagonal matrix. This works for almost all matrices A. Even for those for which this doesn't work, you can always write $PA = LU$ for a suitable permutation matrix P.

1. Show that for any matrix A, the square matrix $S = A^T A$ is symmetric. For any vector $\boldsymbol{v}$, show that:

$$\boldsymbol{v}^T \underbrace{A^T A}_{S} \boldsymbol{v} \tag{1}$$

is a (1×1 matrix whose only entry is a) non-negative number.

Solution:

2. Compute the inverse of the matrix:

$$A = \begin{bmatrix} 1 & 6 & -1 \\ 3 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

by Gauss-Jordan elimination on the augmented matrix $\big[A|I\big]$.

Solution:

3. Compute the $PA = LDU$ factorization of the matrix:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$$

Solution:

4. Write down the 4×4 matrices corresponding to the permutation $\{2, 1, 4, 3\}$ and $\{2, 3, 4, 1\}$ and compute their product. Is the product also a permutation matrix, and if so, to which permutation does it correspond?

Solution: