Recitation 1. September 10

Focus: rules of matrix multiplication; Gauss-Jordan elimination is LU factorization.

Remark. The most basic rule that you should remember: $\boxed{\textbf{row column}}$. It shows the order in which you write or compute, e.g.:

- The first index denotes the row, the second number the column.
- You multiply a row by a column to get a number.
- An $n \times m$ matrix has n rows and m columns.

Remark. The formula $\boxed{\text{left matrix multiplication corresponds to row operations}}$ explains the mathemagic behind Gauss-Jordan elimination. More precisely, performing row operations on a matrix A is the same as doing LA for some other matrix L.

LU factorization of a matrix A is a way of writing A as a product of two matrices $\boxed{A = LU}$, where L is a lower-diagonal matrix with units on the diagonal and U is an upper-diagonal matrix.

1. *Rules of matrix multiplication. (Section 2.4 of Strang.)* Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \end{bmatrix}, \ B = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix}, \ C = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \ D = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Which of these matrix operations are allowed?

a) AB
b) $(A+B)C$
c) $C(A+B)$
d) AD
e) DA
f) CAD

2. *Binomial formula for matrices.* Show that $(A+B)^2$ is different from $A^2 + 2AB + B^2$ when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule: $(A+B)^2 = A^2 + \cdots + B^2$.

Solution:

3. Consider the following matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \ B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ D = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}.$$

How is each row of BA, CA, DA related to the rows of A?

Solution:

4. *LU factorization = Gaussian elimination.* Solve the system of linear equations using LU factorization:

$$\begin{cases} x + 2y + 3z = 1, \\ \quad\quad y + \ \ z = 2, \\ 3x + \ \ y - \ \ z = 3. \end{cases}$$

Solution:

5. *Not all matrices can be written in LU form. (You will deal with this kind of situation when discussing $PA = LU$ factorizations on Friday.)* Show directly why this matrix equation is impossible:

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l & 1 \end{bmatrix} \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}.$$

Solution: