Course 18.06: Problem Set 4

Due 4PM, Thursday 15th October 2015, in the boxes at E17-131.

This homework has 5 questions to hand-in. Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and clearly PRINT your name, recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online self-graded part.

Problems 1–5

1. [10 pts] Let rref(A) denote the reduced row echelon form of A. For $n \times n$ square matrices A and B, is it true that

$$\text{rref}(AB) = \text{rref}(A)\text{rref}(B)?$$

If true, then give an argument. If false, then give a counterexample.

2. [10 pts] Give a basis for the column space, row space, null space, and left null space of A and B, where

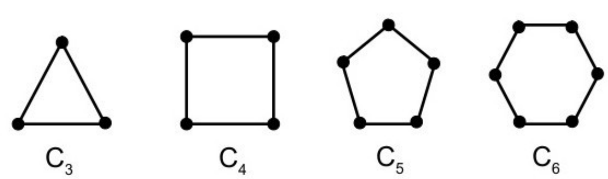
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 1 & 2 & -2 & 3 \\ -2 & 1 & 3 & 0 \end{bmatrix}.$$

3. [10 pts] Let A be a 6×6 adjacency matrix for a simple (but unspecified) graph with 6 nodes. A graph is *simple* if any two nodes are connected by at most one edge, and there are no loops (no edges connecting a node to itself). Consider these three quantities:

$$\sum_{i=1}^{6} A_{ii}, \qquad \sum_{i=1}^{6} (A^2)_{ii}, \qquad \sum_{i=1}^{6} (A^3)_{ii},$$

where $(A)_{ii}$ is the (i,i) entry of A, $(A^2)_{ii}$ is the (i,i) entry of A^2, and $(A^3)_{ii}$ is the (i,i) entry of A^3. What is each one counting in terms of the graph? (Maybe try small graphs and spot patterns.)

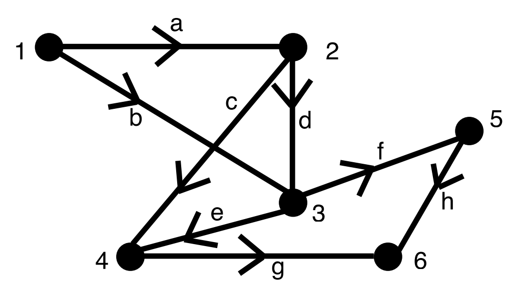
4. [10 pts] Let C_n be the cycle with n nodes (so its edges are $(1,2), (2,3), \ldots, (1,n)$). For example, here are C_3, C_4, C_5, and C_6:



(a) Are the adjacency matrices of C_3, C_4, C_5, and C_6 invertible?

(b) By solving $C_n \underline{x} = \underline{0}$ for $\underline{x}$, show that C_n is not invertible if n is a multiple of 4. (Maybe try $n = 4$ and $n = 8$ first.)

5. [10 pts] Consider the directed graph with six nodes (numbered) and eight edges (lettered):



(a) Write down the 8×6 incidence matrix A.
(b) (i) Give one vector x such that $Ax = 0$.
(ii) Describe the vectors w such that $A^T w = 0$.
(iii) Using the graph, why is $A^T w = 0$, where $w = (0, 0, 1, -1, 0, -1, 1, -1)^T$.