Course 18.06: Problem Set 3

Due 4PM, Thursday 8th October 2015, in the boxes at E17-131.

This homework has 4 questions to hand-in. Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and clearly PRINT your name, recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online self-graded part.

Problems 1–4

1. [10 pts] What is the null space of A, when

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}?$$

Label the free-variables, the pivot columns, and show your row-reduced echelon form of A.

(b) Write down all the solutions to $Ax = b$, where $b = \begin{bmatrix} 1 & 1 & 4 \end{bmatrix}^T$.

(c) For general A and b, how many solutions does $Ax = b$ have? Complete the four spaces in the table below:

	$N(A)$ only contains the zero vector	$N(A)$ contains more than the zero vector
$b \in C(A)$		
$b \notin C(A)$		

Here, $C(A)$ denotes the column space of A and $N(A)$ denotes the nullspace of A.

2. [10 pts]
(a) Give a basis for the vector space of upper triangular matrices.
(b) Give a basis for the vector space of symmetric matrices.
(c) Is there a basis for the collection of permutation matrices?

3. [15 pts] Are the following statements TRUE or FALSE? (Note that the *row space* of A is the same as the column space of $C(A^T)$.)
(a) If B is an echelon form of A, and if B has two nonzero rows, then the first two rows of A form a basis for the row space of A.
(b) If A is not square, then the dimensions of the row space and column space of A are the same.
(c) On a computer row operations can change the rank of a matrix.

4. [15 pts] Let A be given by

$$A = \begin{bmatrix} -2 & 4 & 6 \\ 2 & -4 & 3 \\ 1 & -2 & 1 \end{bmatrix}.$$

Find a basis for $C(A)$ and $N(A)$.

(b) Explain why $\dim(C(A)) + \dim(N(A)) = 3$.

(c) If A is an $m \times n$ matrix, then $\dim(C(A)) + \dim(N(A))$ always equals n. Using elimination and the echelon form (or anything else), can you explain why? Try 3×3 examples if you wish. (We are not asking for a water-tight proof, only an explanation.)