# Course 18.06: Problem Set 2

Due 4PM, Thursday 24th September 2015, in the boxes at E17-131.

This homework has 4 questions to hand-in. Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and clearly PRINT your name, recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online self-graded part.

## Problems 1–4

1. [15 pts] (a) Determine if the matrices $A$, $B$, and $C$ are invertible. If the matrix is invertible, then find its inverse by Gauss–Jordan. For $D$ find the inverse in terms of $a$.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad D = \begin{bmatrix} a & 1 & 1 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}.$$

(b) For what values of $a$ is $D$ invertible? How does your formula for $D^{-1}$ breakdown for values of $a$ for which $D$ is not invertible?

2. [5 pts] A company has three ground water sources. Each source is used to produce a combination of tap water, bottled water, and industrial-grade water used for agricultural irrigation. From 1 barrel of ground water, source 1 produces 10 gallons of tap water, 30 gallons of bottled water, and 50 gallons of water for industry. If one barrel of ground water is used, the output from the other sources are as follows:

| | Source 1 | Source 2 | Source 3 |
|---|---|---|---|
| Tap water | 10 | 20 | 10 |
| Bottle water | 30 | 70 | 20 |
| Industrial-grade water | 50 | 100 | 60 |

Suppose that we have a daily demand for 240 gallons of tap water, 670 gallons of bottle water, and 1280 gallons of industrial-grade water. How many barrels of ground water are needed at each source so that the total output of the 3 sources **exactly** satisfies the daily demand? Please solve by hand.

3. [15 pts] Are the following statements true or false? For each statement explain why it is true or give a counterexample if it is false. Recall that a matrix $A$ is symmetric if $A^T = A$.
(a) The product of two upper triangular matrices is an upper triangular matrix.
(b) Upper triangular matrices form a subspace in the vector space of all matrices.
(c) The product of two permutation matrices is a permutation matrix.
(d) Permutation matrices form a subspace in the vector space of all matrices.
(e) The product of symmetric matrices is a symmetric matrix.
(f) Symmetric matrices form a subspace in the vector space of all matrices.

4. [15 pts] "In principle, one can avoid temporary failures by reordering the equations at the start"
(a) Give a $3 \times 3$ matrix $P$ so that $PA$ swaps the last two rows of the $3 \times 3$ matrix $A$ and leaves the first row unchanged.
(b) Out of interest, what do $AP$, $PAP$, and $P^2A$ do to $A$?
(c) Use elimination (and fix the temporary failure) to find the elimination matrices $E_{21}$, $E_{31}$, and $E_{32}$ and an upper-triangular matrix $U$ so that

$$E_{32}PE_{31}E_{21}\underbrace{\begin{bmatrix} 3 & 4 & 1 \\ 3 & 4 & 2 \\ 1 & 0 & 1 \end{bmatrix}}_{=A} = U.$$

(d) Show that $E_{32}PE_{31}E_{21} = E_{32}(PE_{31}P)(PE_{21}P)P$. (Note that $PE_{31}P$ and $PE_{21}P$ look like elimination matrices.)
(e) Explain why (d) means that one can avoid temporary failures altogether by reordering the rows of $A$ at the start.
(f) Explain why a computer does not reorder the equations at the start to prevent temporary failure.
(g) Calculate the $PA = LU$ factorization of $A$.