Solutions to pset 1

① $$1\begin{bmatrix}2\\4\\4\end{bmatrix} + 1\begin{bmatrix}-3\\-6\\3\end{bmatrix} + 0\begin{bmatrix}4\\10\\10\end{bmatrix} = \begin{bmatrix}-1\\-2\\7\end{bmatrix}$$

(Many ways to come up with this answer.)

② a) $y = 1 \quad z = 0$

b) $y = 2 \quad z = -1$

c) If $A\begin{bmatrix}y\\z\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ then (by eq. 1) $y = -2z$.

$\therefore$ If $z = -\beta$, where β is some number,

then $y = 2\beta$.

(Do not accept $A\left(\beta\begin{bmatrix}2\\-1\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix}$ ... wrong math logic.)

d) $$A\begin{bmatrix}y+8\\z-4\end{bmatrix} = A\begin{bmatrix}y\\z\end{bmatrix} + 4A\begin{bmatrix}2\\-1\end{bmatrix} = A\begin{bmatrix}y\\z\end{bmatrix} = b$$

3. $$M_1 = \begin{bmatrix}2 & -1 & 3\\2 & -1 & 3\\2 & -1 & 3\end{bmatrix},\quad M_2 = \begin{bmatrix}0 & 0 & 0\\0 & 2 & -1\\0 & -6 & 3\end{bmatrix},\quad M_3 = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 4\end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 1 & 2 \\ 2 & -7 & 10 \end{bmatrix}$$

Elimination on A:

$$\begin{bmatrix} 2 & -1 & 3 \\ 2 & 1 & 2 \\ 2 & -7 & 10 \end{bmatrix} \xrightarrow[\text{③}=\text{③}-\text{①}]{\text{②}=\text{②}-\text{①}} \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & -6 & 7 \end{bmatrix} \quad \text{③}=\text{③}+3\text{②}$$

$$\longrightarrow \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

Same operations on b:

$$\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 4 \\ 12 \end{bmatrix}$$

Now solve:

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 12 \end{bmatrix}$$

$$\Rightarrow \boxed{z = 3}, \quad 2y = 4 + z \Rightarrow \boxed{y = 7/2}$$

$$2x = 1 + y - 3z = -9/2$$

$$\Rightarrow \boxed{x = -9/4}$$

4) I pick (OTHER EXAMPLES POSSIBLE)

(a) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

then $AB = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ $BA = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$, $B^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ then $A^2 = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) Look at diagonal sums $tr(A) = \sum a_{11} + a_{22}$

then

$$tr(AB - BA) = tr(AB) - tr(BA)$$
$$= tr(AB) - tr(AB) = 0$$

But $tr(I) = 2$

OR just try with $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$