Course 18.06: Problem Set 1

Due 4PM, Thursday 17th September 2015, in the boxes at E17-131.

This homework has 4 questions to hand-in. Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and clearly PRINT your name, recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

This homework also has an online self-graded part. Either in Julia (follow the instructions on the course website) or in MITx and MATLAB (go to `lms.mitx.mit.edu`). We are encouraging Julia — it is fast, free, and very useful.

Problems 1–4

1. [10 pts] Express $b = \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ -6 \\ 3 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 4 \\ 10 \\ 10 \end{bmatrix}$.

2. [10 pts] (a) Find any numbers y and z such that

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}}_{=A} \begin{bmatrix} y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{=b}.$$

(b) Now find numbers y and z such that $A \begin{bmatrix} y \\ z \end{bmatrix} = 0$.

(c) If $A \begin{bmatrix} y \\ z \end{bmatrix} = 0$, then explain why we must have $\begin{bmatrix} y \\ z \end{bmatrix} = \beta \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ for some number β.

(d) Take the numbers y and z from part (a). Without any calculation, explain why $A \begin{bmatrix} y+8 \\ z-4 \end{bmatrix} = b$.

3. [15 pts] (a) Compute the 3 by 3 matrices M_1, M_2, M_3 where

$$M_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 0 & 2 & -1 \end{bmatrix} \quad \text{and} \quad M_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 4 \end{bmatrix}$$

(b) Use elimination to solve $Ax = b$, where $A = M_1 + M_2 + M_3$ and $b = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$.

4. [15 pts] "The algebra of matrices is different from the algebra of numbers". Find examples of 2×2 matrices (different examples for each part) such that:
(a) $AB \neq BA$,
(b) $B^2 = 0$ and B has no zero entries,
(c) $A^2 = -I$, where A has real entries and I is the identity matrix,
(d) But, show that $AB - BA = I$ is impossible for square 2×2 matrices A and B.