# 18.06, SVD practice

(1) Find the SVD of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

(2) Find the SVD of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

(3) Find the singular value(s) of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & \pi & 6 \end{bmatrix}.$$

(4) If $A \in \mathbb{R}^{n\times n}$ and $A = U\Sigma V^T$, then pick the largest constant $c_1 \in \mathbb{R}$ and smallest constant $c_2 \in \mathbb{R}$ so that the following inequality holds for all vectors $\underline{x} \in \mathbb{R}^n$:

$$c_1\|\underline{x}\| \leq \|A\underline{x}\| \leq c_2\|\underline{x}\|.$$

Give one vector that attains the lower bound and one vector that attains the upper bound.

(5) Find the SVD of the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

From the SVD, give an orthonormal basis for the four fundamental subspaces of $A$: $C(A)$, $N(A)$, $C(A^T)$, and $N(A^T)$.

(6) Write down the SVD of the following matrices:
  (i) Orthogonal matrix $Q$,
  (ii) Diagonal matrix $D$,
  (iii) Symmetric matrix $A$,

(7) Let $A$ be a positive definite matrix. Explain the connection between the eigenvalue decomposition and the singular values decomposition of $A$. Is there a connection if $A$ is negative definite?