Exam 1 Review
Solutions

1. "… $C(A) = \underline{\mathbb{R}^n}$ … $N(A) = \underline{\left\{ \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \right\}}$ … pivot colums are $\underline{1 \text{ to } n}$ …

solution to $Ax = b$ is $\underline{\text{unique}}$ $\underline{(\text{also accept } A^{-1}b)}$ …"

2. $$C(A) = c\begin{bmatrix} 1 \\ 2 \end{bmatrix} + d\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \mathbb{R}^2$$

$$N(A) = c\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

pivot cols = First two, col 1 & col 2.

Solution to $A\underline{x} = \underline{b}$ is $\underline{\text{not}}$ unique (also allow: $\begin{bmatrix} -b_2+3b_2 \\ b_2-2b_1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

For special solutions see $N(A)$ above.

3. $$C(A) = c\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$N(A) = c\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

pivot cols = ~~1st 2 and~~ 1st and 2nd

Sol to $A\underline{x} = \underline{b}$ is $\underline{\text{not}}$ unique and $\underline{\text{only}}$ exists if $\underline{b} \in C(A)$

if $b \in C(A)$, then ~~#~~ sol^n to $A\underline{x} = \underline{b}$ are $\begin{bmatrix} b_1 \\ 0 \\ b_2-b_1 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

For special solutions, see $N(A)$ above.

4 a) $b \in C(A)$ if and only if $E\underline{b} \in C(EA)$

$N(A)$ and $N(EA)$ are the same.

5. Five equations and 6 unknowns $\Rightarrow$ at least one free variable.

(b) $C([A\ A])$ is the same as $C(A)$

(c) There are at least $7\ (=12-5)$ special solution to generate $N(A)$.

6. There are only $n!$ permutation matrices of size $n \times n$. Consider the $n!+1$ matrices

$$I,\ P,\ P^2,\ \dots,\ P^{n!-1},\ P^{n!}$$

All ~~P~~ $n \times n$ permutation matrices. Since there ~~a~~ are $n!$ permutation matrices we must have $P^{n!} = P^k$ for some k. That is, $P^{n!-k} = I$ for some k.

7. AB invertible so $I = (AB)(AB)^{-1} = A\left(B(AB)^{-1}\right)$

Since A is square and so is $B(AB)^{-1}$, $\underline{A^{-1} = B(AB)^{-1}}$.

(b) let A be 5×4, and B be 4×5. Consider AB.

Since A is 5×4, $C(A)$ is $\underline{\text{not}}$ $\mathbb{R}^5$. (Can only contain 4 "good" vectors in $C(A)$.

Therefore, $C(AB)$ is $\underline{\text{not}}$ $\mathbb{R}^5$ as $C(AB)$ is a subspace of $C(A)$.

if $(AB)^{-1}$ exists then $C(AB)$ is $\mathbb{R}^5$. Since it cannot be, $(AB)^{-1}$ does not exist.

8. Assume A is an $\underline{n \times n \text{ matrix}}$:

$(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4) \Leftrightarrow (5) \Leftrightarrow (6) \Leftrightarrow (7)$

~~Assume A is an $m \times n$ matrix with $m > n$~~:

~~$(1) \Rightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4) \Leftrightarrow$~~

~~B~~ Think about the echelon form for reasoning.

9, 10, 11, 12 $\Rightarrow$ solutions online.