

## 18.06 Exam 1 review

1. Fill the blanks: for an  $n \times n$  invertible matrix  $A$ , the column space  $C(A) = \text{-----}$ , the null space  $N(A) = \text{-----}$ , the pivot columns are -----, and the solution to  $A\mathbf{x} = \mathbf{b}$  is -----.
2. Answer to the same question as in Problem 1, when  $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \end{bmatrix}$ . What are the special solutions to  $A\mathbf{x} = \mathbf{0}$ ?
3. Answer to the same question as in Problem 1, when  $A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ . What are the special solutions to  $A\mathbf{x} = \mathbf{0}$ ?
4. If  $E$  is a square and invertible matrix,
  - (a) How is  $C(EA)$  related to  $C(A)$ ?
  - (b) How is  $N(EA)$  related to  $N(A)$ ?
5. If  $A$  is a  $5 \times 6$  matrix,
  - (a) Why are there nonzero solutions to  $A\mathbf{x} = \mathbf{0}$ ?
  - (b) How is  $C([A \ A])$  related to  $C(A)$ ? (Note that  $[A \ A]$  is a  $5 \times 12$  matrix.)
  - (c) There are at least -- “special” solutions to  $[A \ A] \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{0}$  that generate  $N(A)$  from linear combinations.
6. If  $P$  is a permutation matrix, explain why  $P^N = I$  holds for some positive integer  $N$ .
7.
  - (a) If  $A$  and  $B$  are  $4 \times 4$  and  $AB$  is invertible, show that  $A$  is invertible.
  - (b) A  $5 \times 4$  matrix times a  $4 \times 5$  matrix cannot produce an invertible  $5 \times 5$  matrix. Why not?
8. Are the following 8 statements are equivalent? Explain.
  - (1) The rank of  $A$  is  $n$
  - (2) All the columns of  $A$  are pivot columns
  - (3) The nullspace  $\mathbf{N}(A)$  contains only the zero vector
  - (4) The space  $\mathbf{C}(A^T)$  is all of  $\mathbf{R}^n$
  - (5) The columns of  $A$  are a basis for its column space
  - (6) If  $Ax = Ay$  then  $x = y$  (uniqueness of solutions to  $Ax = b$ )
  - (7) The matrix  $A^T A$  is invertible (and symmetric)
9. Spring 2014, Exam 1, Problem 2
10. Spring 2014, Exam 1, Problem 3
11. Fall 2012, Exam 1, Problem 1
12. Two good final practice sets are Exam 1 from Fall 2014 and Spring 2015. Remember you only have 50min to do it.