

Let A be the matrix below

```
In [1]: A=[ 5   9  -4   3   4;
          -2  -6   0  -9  -4;
           1   2  -2  -8  -7;
           2  -5  -8   5  -7;
          -6  -7   1  -3  -3]
```

1. Consider the following functions

$$f(x) = \exp(x)$$

$$f(x) = 2/x + 3 + 5x^3$$

$$f(x) = x \cdot \exp(x)$$

For each function compute a matrix that has the same eigenvectors as A, but whose eigenvalues are $f(\lambda)$ where λ is an eigenvalue of A. Verify that the eigenvalues are as you expect. Make sure you understand what is going on. Note that in Julia and MATLAB, `expm` is the matrix exponential, while `exp` is the element-wise exponential.

Turn in your eigenvalue computation and the function applied to the eigenvalues:

1. The third function $f(x) = x \cdot \exp(x)$ is the derivative of $\exp(tx)$ with respect to t , at $t=1$. What is the corresponding matrix derivative? Do a simple finite difference to compute the matrix derivative that is accurate to three or four places.

```
In [2]:  $\lambda$ =eigvals(A) # Note: You can type \lambda<tab> for the character  $\lambda$ 
```

```
In [3]: s=v->sort(v,by=real) # create a simple sort_by_real_part function
```

```
In [4]: #1. f(x)=exp(x)
[eigvals(expm(A)) exp( $\lambda$ )]
```

```
In [5]: #sorting by real part makes it easier to see
[s(eigvals(expm(A))) s(exp( $\lambda$ ))]
```

```
In [6]: #2. f(x) = 2/x + 3 + 5x^3
[s(eigvals(2*inv(A) + 3*I + 5*A^3)) s(2./ $\lambda$ +3+5* $\lambda$ .^3)]
```

```
In [7]: #3. f(x)= x*exp(x)
[s(eigvals(A*expm(A))) s( $\lambda$ .*exp( $\lambda$ ))]
```

```
In [8]: A*expm(A)
```

```
In [9]: #Finite Difference Approximation to d/dt exp(At) at t=1  
h=.000001  
t=1  
(expm(A*t*(1+h))-expm(A*t))/h
```

```
In []:
```