

The purpose of this assignment is to show that one can obtain the Legendre Polynomials numerically from the QR decomposition

The Legendre polynomials are orthogonal on $[-1,1]$ and have many applications. Please see http://en.wikipedia.org/wiki/Legendre_polynomials (http://en.wikipedia.org/wiki/Legendre_polynomials)

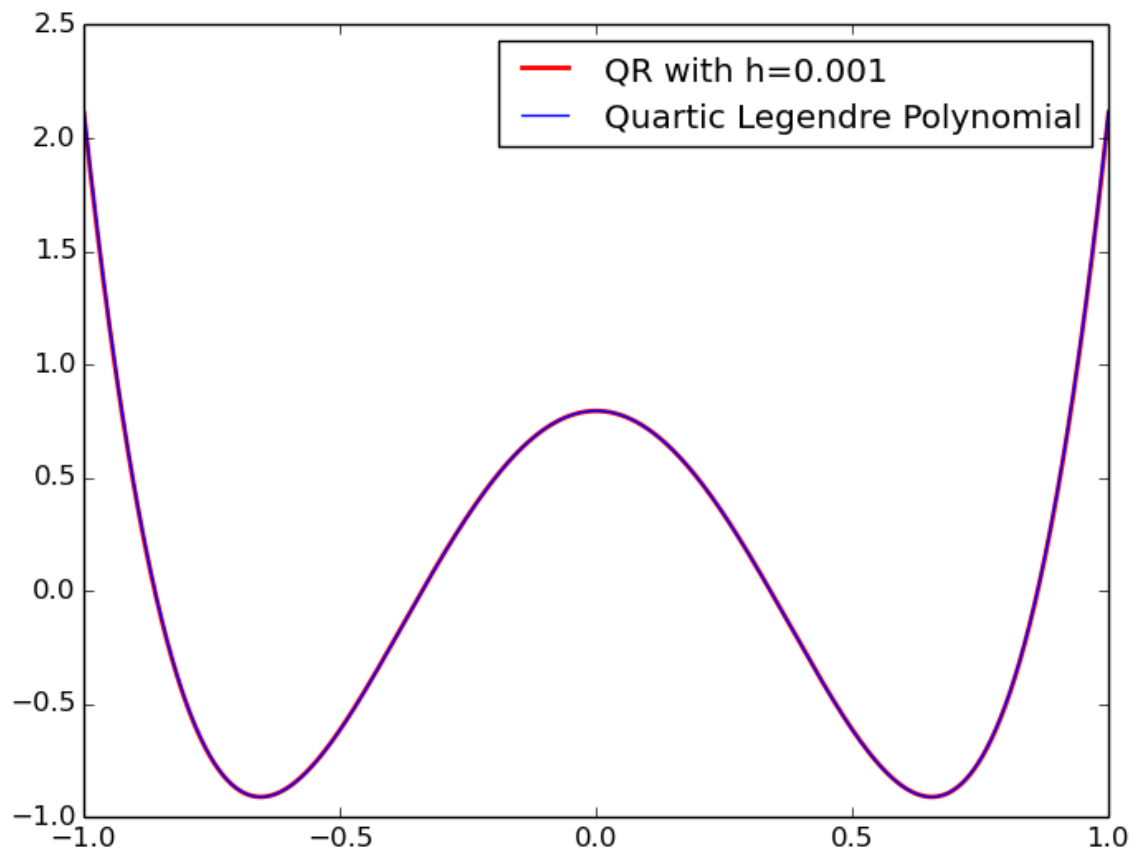
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In [1]: # Let x be a vector of equally spaced points from -1 to 1
# Let the columns of A be the powers of x
# One can think of A as having first column 1, second column x,
# third column x^2, etc.
N = 1000
h = 1/N
k = 5
A = zeros(2*N+1,k+1)
x = -1: 1/N : 1
for i=1:2N+1,j=1:k
    A[i,j] = x[i]^(j-1)
end
```

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In [5]: # Compute the QR decomposition of A (Gram-Schmidt)
(q,r) = qr(A)
q=q/sqrt(h); # Normalize q to integrate to 1;
```

You are asked to compare graphically with the first five columns of Q with the Legendre polynomials to see if they line up Note: You must compare with the nth Legendre polynomial times $\sqrt{n+5}$. Do you see why? Note that it is possible you may be off by a factor of -1. Can you explain why?

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In [23]: using PyPlot;
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In [22]: plot(x, -q[:,5], linewidth=2,"r")
plot(x, (35*x.^4 - 30*x.^2 + 3)/8 * sqrt(4.5))
legend({"QR with h=$h", "Quartic Legendre Polynomial"})
```



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Out[22]: PyObject <matplotlib.legend.Legend object at 0x7fe61c0d1c10>
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Please submit pictures up to the quartic like the one above and answer the two questions.

In []: