18.06 Problem Set 6

due Thursday, October 23, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

Problem 1. Section 4.4, Problem 5, page 239.

Problem 2. Section 4.4, Problem 10, page 240.

Problem 3. Section 4.4, Problem 17, page 241.

Problem 4. Section 4.4, Problem 23, page 242.

Problem 5. Section 5.1, Problem 3 page 251.

Problem 6. Section 5.1, Problem 18 page 253.

See next page

Problem 7. Consider the vector space of functions $f(x)$ with the inner product $(f, g) = \int_0^{2\pi} f(x)\, g(x)\, dx$, as described on page 448 of the textbook.

One can use exactly the same Gram-Schmidt process for this space of functions as for $\mathbb{R}^n$, where, instead of the dot-product of vectors in $\mathbb{R}^n$, one needs to use the inner product of functions.

(a) Consider the subspace of quadratic functions $f(x) = a + bx + cx^2$. Construct an orthogonal basis of this subspace by using Gram-Schmidt for the basis 1, x, x^2.

(b) Construct an orthonormal basis of this subspace.

Problem 8. Consider the family of "almost upper-triangular" matrices A_n:

$$A_1 = (1),\ A_2 = \begin{pmatrix} 1 & 1 \\ x_1 & 1 \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 1 & 1 \\ x_1 & 1 & 1 \\ 0 & x_2 & 1 \end{pmatrix},\ A_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ x_1 & 1 & 1 & 1 \\ 0 & x_2 & 1 & 1 \\ 0 & 0 & x_3 & 1 \end{pmatrix},$$

$$A_5 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ x_1 & 1 & 1 & 1 & 1 \\ 0 & x_2 & 1 & 1 & 1 \\ 0 & 0 & x_3 & 1 & 1 \\ 0 & 0 & 0 & x_4 & 1 \end{pmatrix},\ A_6 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ x_1 & 1 & 1 & 1 & 1 & 1 \\ 0 & x_2 & 1 & 1 & 1 & 1 \\ 0 & 0 & x_3 & 1 & 1 & 1 \\ 0 & 0 & 0 & x_4 & 1 & 1 \\ 0 & 0 & 0 & 0 & x_5 & 1 \end{pmatrix},\ \dots$$

The matrix A_n is the $n \times n$ matrix with 1's on the main diagonal and everywhere above it, with $x_1, x_2, \dots, x_{n-1}$ on the diagonal immediately below the main diagonal, and with 0's everywhere below it. (Here $x_1, x_2, \dots$ are variables, which are not equal to 1.)

(a) Use row operations to find the pivots of the matrices A_1, A_2, A_3. Calculate the determinants of these matrices.

(b) Calculate the determinants of A_4, A_5, A_6. (You may use your favorite computational software.) Express these determinants as products of simple factors. Can you guess an expression for $\det(A_n)$ for any n?

(c) Prove your guess for $\det(A_n)$ by calculating all pivots of A_n.

Problem 9. It is not true in general that $\det(A + B) = \det(A) + \det(B)$.

(a) Find an example of matrices A and B such that $\det(A+B) \neq \det(A)+\det(B)$.

(b) Find an example of 2×2 matrices A and B such that $\det(A+B) = \det(A) + \det(B)$ and all 3 determinants are nonzero.

Problem 10. (Computational Problem)

Available at `http://web.mit.edu/18.06/www/Fall14/ps6c.pdf`