18.06 Problem Set 5

due Thursday, October 16, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

You can use your favorite computational software in any problem. Of course, you should not use a computer program that, for example, does least square approximation if your problem is to find the least square approximation. You need to set up all equations youself. But you may use a computer to multiply numbers and matrices, invert matrices, etc.

Problem 1. Section 4.1, Problem 29, page 205

Clarification: A matrix has four fundamental subspaces, and these can be made into 6 pairs. Which of these 6 pairs can v not be in?

Problem 2. Section 4.2, Problem 5, page 214.

Problem 3. Section 4.2, Problem 14, page 215.

Problem 4. Section 4.2, Problem 17, page 215.

Problem 5. Section 4.2, Problem 23, page 216.

Problem 6. Section 4.3, Problem 10, page 227.

Problem 7. Section 4.3, Problem 12, page 228.

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Problem 8. Is it true that all square matrices P such that $P^2 = P$ are projection matrices? Prove this for 2×2 matrices or construct a counterexample. (Hint: A projection matrix must be symmetric $P^T = P$.)

Problem 9. Find the exponential function $f(t) = Ae^t$ that fits the three points $(0, 25)$, $(1, 56)$, $(2, 130)$ as close as possible using the least squares approximation, that is, find A that minimizes the sum of squares of errors:

$$|f(0) - 25|^2 + |f(1) - 56|^2 + |f(2) - 130|^2.$$

Use computer or calculator to round your answer to two decimal places.

Problem 10. (Computational Problem)

Available at `http://web.mit.edu/18.06/www/Fall14/ps5c.pdf`