18.06 Computational PSet 4

You may use any computer language. Please submit printouts with the problem set.

1. This problem asks you to solve for voltages in currents following the methodology of Example 1 on page 427.

Set up the incidence matrix for the graph in Figure 8.5:

In[1]:
```
A=[-1 -1 0 -1 0 0 ; 1 0 -1 0 -1 0 ;0 1 1 0 0 -1; 0 0 0 1 1 1]'
```

Out[1]:
```
6x4 Array{Int64,2}:
-1   1   0  0
-1   0   1  0
 0  -1   1  0
-1   0   0  1
 0  -1   0  1
 0   0  -1  1
```

Now write a function that inputs six conductance values in a vector `c`
and 3 current sources in a vector `f` and solves for the voltages and currents. Explain briefly how the code works.

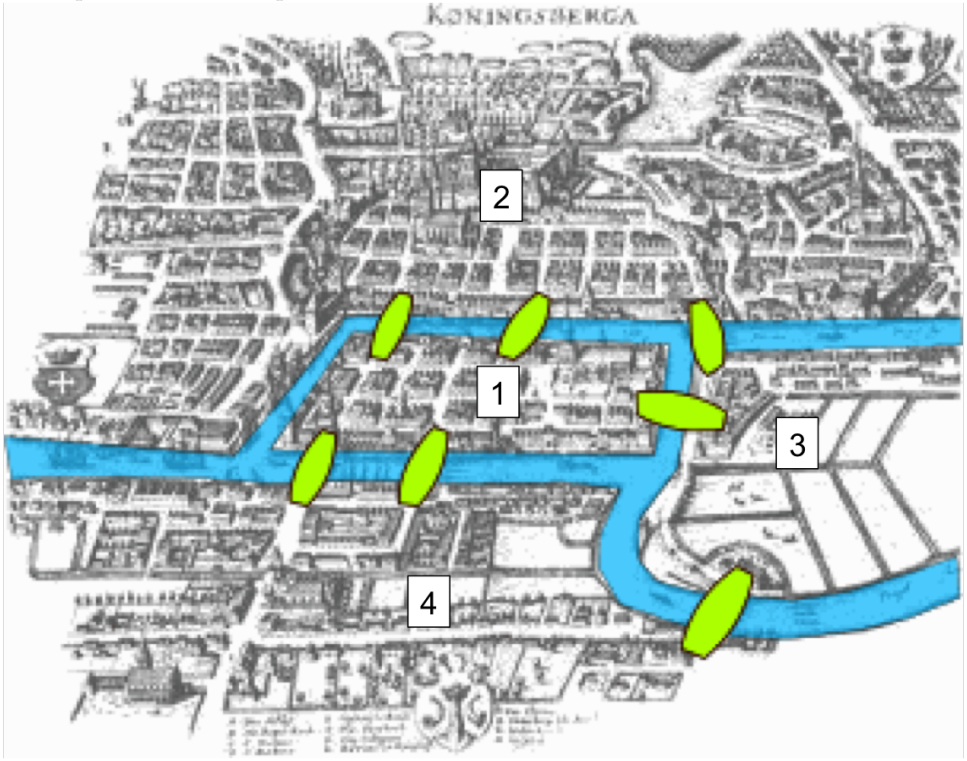
In[2]:
```
function network(c,f)
  A0=A[:,1:3]
  C=diagm(c) # diagonal matrix
  V = (A0'*C*A0)
  y = -C*A0*V # Correction
 (V,y)
end
```

Try running this code with

In[3]:
```
c=[0,1,1,1,1,1]
f=[8,0,0]
```

and write down the currents and voltages you get. Change c[1] to 1 and again write down the currents and voltages. Finally let c[1] $\to \infty$ and write down the observed currents and voltages.

2. There is a famous historical graph theory problem known as the seven bridges of Königsberg. Here is a picture from wikipedia:



There are four regions and the number of bridges between region i and region j may be tabulated in a matrix:

```
In[4]:  A=[0 2 1 2; 2 0 1 0; 1 1 0 1; 2 0 1 0]
```

```
Out[4]:  4x4 Array{Int64,2}:
         0  2  1  2
         2  0  1  0
         1  1  0  1
         2  0  1  0
```

It is known that the [i,j] element of A^k counts the number of paths crossing from region i to region j crossing exactly k bridges.

a) Compute A^k for $k = 0, 1, 2, 3$. Find the four paths involving two bridges from region 1 to region 3.

b) How many paths from the center island (region 1) crossing 10 bridges will end up back on the center island?