18.06 Problem Set 4

due Thursday, October 8, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

Problem 1. Section 3.5, Problem 16, page 180.

Problem 2. Section 3.5, Problem 26, page 181.

Problem 3. Section 3.6, Problem 5, page 191.

Problem 4. Section 3.6, Problem 27, page 194.

Problem 5. Section 8.2, Problem 8, page 429.

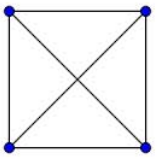
Problem 6. Section 8.2, Problem 10, page 429.

Problem 7. Section 4.1, Problem 17, page 204.

Problem 8. Section 4.1, Problem 22, page 204.

See next page.

Problem 9. The **complete graph** K_4 is the graph that contans 4 vertices, and any pair of vertices is connected by an edge:

$K_4 =$ 

(a) Show how to pick an ordering of the vertices and edges of this graph and direct arrows on the edges, so that the (signed) incidence matrix becomes

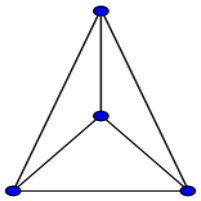
$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

(b) Which subgraphs of this graph are spanning trees? (A **spanning tree** is a subgraph without cycles that contains all vertices of the graph.)

Which subsets of rows of A are bases of the row space $C(A^T)$?

(c) Find the dimensions of the 4 fundamental subspaces of A.

(d) We can draw the complete graph K_4 on the plane in a different way so that no pair of edges cross each other:

$K_4 =$ 

(Graphs that can be drawn on the plane without crossing edges are called **planar graphs**.)

Follow the discussion on pages 425–426 of the textbook and explain how the 3 little triangles in this figure give you a basis of the left nullspace $N(A^T)$. Explain how the formula $\dim N(A^T) = m - r$ produces **Euler's formula** for planar graphs.

(e) Find bases of the 4 fundamental subspaces of A.

(f) Calculate the **Laplacian matrix** $L = A^T A$.

(g) Find the dimensions and bases of the 4 fundamental subspaces of the Laplacian matrix $L = A^T A$.

Problem 10. (Computational Problem)

Available at

`http://web.mit.edu/18.06/www/Fall14/ps4c.pdf`