18.06 Problem Set 3

due Thursday, September 25, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

Problem 1. Section 3.2, Problem 9, page 141.

Problem 2. Section 3.2, Problem 12, page 141.

Problem 3. Section 3.2, Problem 27, page 142.

Problem 4. Section 3.3, Problem 2, page 152.

Problem 5. Section 3.3, Problem 10, page 152.

Problem 6. Section 3.4, Problem 1, page 163.

Problem 7. (a) Find a 3×3-matrix whose nullspace is the line spanned by the vector $(3\ 0\ 1)^T$.

(b) Find a 3×3-matrix whose nullspace is the plane spanned by the vectors $(3\ 0\ 1)^T$ and $(1\ 1\ 2)^T$.

Problem 8. Let a, b, c, d, e be five nonzero numbers, and let

$$A = \begin{pmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{pmatrix}.$$

(a) Find the RREF for the matrix A.

(b) What is the rank of A?

(c) Find the special solution(s) of $A\mathbf{x} = (0\ 0\ 0)^T$.

(d) Find the complete solution of $A\mathbf{x} = (0\ 1\ 0)^T$.

(e) For which values of the parameters a, b, c, d, e does the vector $(1\ 0\ -1)^T$ belong to the nullspace of A?

Problem 9. Calculate the ranks of the following matrices. Are these matrices invertible?

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,

(b) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$,

(c) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$,

(d) $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$,

(e) $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$,

(f) $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$,

Problem 10. (Computational Problem)   Available at

`http://web.mit.edu/18.06/www/Fall14/ps3c.pdf`