18.06 Problem Set 2

due Thursday, September 18, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

Problem 1. Section 2.6, Problem 13, page 104.

Problem 2. Section 2.6, Problem 16, page 105.

Problem 3. Section 2.6, Problem 19, page 105.

Problem 4. Section 2.7, Problem 15, page 117.

Problem 5. Section 2.7, Problem 17, page 117.

Problem 6. Section 3.1, Problem 10, page 128.

Problem 7. Find a symmetric matrix whose column space is the line that contains the vector $\mathbf{v} = (1, 2, 3)^T$.

For the following Problems 8, 9, and 10 you may use your favorite computational software `Julia, Mathematica, Matlab, Maple, Python, Sage`, etc. But, if you prefer, you may solve Problems 8 and 9 without using a computer.

Problem 8. You probably heard about **Pascal's triangle**, which is the triangular array of the **binomial coefficients** $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, see `http://en.wikipedia.org/wiki/Pascals_triangle` for more details.

Let A be the lower triangular 6×6 matrix filled with the first 6 rows of Pascal's triangle:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 \end{pmatrix}$$

Each entry in this matrix (except the first row and the first column) is the sum of the entries immediately above it and immediately to the NW of it.

(a) Calculate $A\,(1,1,1,1,1,1)^T$, $A^2\,(1,1,1,1,1,1)^T$, and $A^3\,(1,1,1,1,1,1)^T$.

(b) Can you guess the general form of $A^m(1,1,1,1,1,1)^T$ for any m? Explain why the answer should have this form.
(Hint: Newton's **Binomial Formula** $(x+1)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$ might be helpful.)

Problem 9. The **order** of a permutation matrix P is the **smallest** positive integer m such that $P^m = I$.

(a) Find 5×5 permutation matrices of order 1, 2, 3, 4, 5, and 6.
(b)* (optional problem) Find a 10×10 permutation matrix of order 30.

Problem 10. (Computational Problem) The problem is available at
`http://web.mit.edu/18.06/www/Fall14/ps2c.pdf`

Attach a printout to your solutions of the problem set.