18.06 Computational PSet 1

You may use any computer language. We encourage trying out Julia. Please submit printouts with the problem set.

a.) Create the $n \times n$ matrix

$$E_n = \begin{pmatrix} 1 & 1 & \dots & 1 & 1 \\ & 1 & 1 & \dots & 1 \\ & & \ddots & \ddots & \\ & & & 1 & 1 \\ & & & & 1 \end{pmatrix}.$$

In some languages this is `triu(ones(n,n))`.

Using the matrix squaring operator create an $n \times n$ "triangular" matrix with 1 on the main diagonal, 2 above, etc.

$$M_n = \begin{pmatrix} 1 & 2 & \dots & n-1 & n \\ & 1 & 2 & \dots & n-1 \\ & & \ddots & \ddots & \\ & & & 1 & 2 \\ & & & & 1 \end{pmatrix}.$$

b.) You very likely have heard of the triangular numbers:

$$T_n = 1 + 2 + \ldots + n = n(n+1)/2.$$

Don't use cumsum, or sum or "+", just matrix products or powers to create the matrix that has the triangular numbers on the diagonals:

$$S_n = \begin{pmatrix} 1 & 3 & \dots & (n-1)n/2 & n(n+1)/2 \\ & 1 & 3 & \dots & (n-1)n/2 \\ & & \ddots & \ddots & \\ & & & 1 & 3 \\ & & & & 1 \end{pmatrix}$$

Explain roughly (not too formal a proof), why your idea works.

c.) Don't stop. Keep going, and get the tetrahedral (see wikipedia) numbers. Explain briefly why this worked.

d.) Let $A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 & 4 \end{bmatrix}$.

Compute $(AB)^{10}$ on the computer. What is AB? What is BA? Explain how it's possible to compute $(AB)^{10}$ without a computer! Hint: $(AB)^{10} = A(BA)^9B$.